Square Roots Teacher Notes

This unit is intended to help students develop an understanding of square roots from a visual / geometric point of view, and also to develop their number sense around this topic. The Common Core State Standards do not mention simplifying radicals, perhaps because that topic has lost its importance in the age of electronic algebra.

Geoboard Area

Because of the visual / geometric emphasis of the unit, it is critical that students have a strong sense of the meaning of area. This lesson helps students grapple with it in a concrete context, by doing simple computations in order to find the areas of figures on the geoboard.

Do not precede this lesson with a review of area formulas. For many of the weaker students, these formulas get in the way of understanding what area even means. However, as students work on the activity, it will probably be useful to discuss the area of rectangles, and the fact that a right triangle can be pictured as half of a rectangle. These ideas will come across most effectively if they arise out of class discussion.

A common misunderstanding in this context is counting pegs to find the side of a rectangle. Make sure to confront this when it arises. This is a conversation that will need to happen one-on-one with many students, but it can also be reinforced by a class discussion.

The activity is taken from *Geometry Labs*. The whole book, including solutions and teacher notes, is available for free download at www.MathEducationPage.org.

If time is short, you may limit the activity to #1-4 and Discussion questions A and B.

Squares and Square Roots

This lesson introduces the central idea of the unit: the square root of a number is the side of a square whose area is that number.

In #1, most students will not find the first seven squares too difficult, but the tilted squares will be considerably more challenging. If students are stumped, or give out-and-out wrong answers, do not reveal the correct answers right away. Instead, suggest that they start by finding the area of the squares. If they need hints on how to do that, you may suggest that they subtract the areas of triangles from the larger enclosing squares. Expressing the length of the sides as $\sqrt{2}$ and $\sqrt{5}$ is sufficient for #1. Decimal approximations are requested in #2.

In #4, the first table can be filled out with whole numbers, but the other two require decimals. You may challenge students to fill out as much as possible without a calculator. Many if not most students will be surprised that in this table, the square roots of numbers are greater than the numbers (except in the case of the total area.) You may have a class discussion of this.

#5 is an extra challenge problem. It is not necessary that every student complete it.

Geoboard Squares

This activity (once again from *Geometry Labs*) follows up on #1 from the previous lesson. Side lengths can be expressed as square roots. (No need for decimal approximations.)

It is not essential that students find all 33 possibilities, but try to make sure that each student has found the area and side length for at least a few tilted squares. If time is short, skip Discussion questions B-D.

On the other hand, if you want to do this lesson in depth, give it a second or even third period, and show the students how it leads to the Pythagorean theorem.

The Exponent ½

This lesson (based on *Algebra: Themes, Tools, Concepts* 9.8) is a review of the laws of exponents, leading to the idea that the positive square root of a number can also be thought of as that number raised to the power ½.

#3-5 will certainly require a class discussion!

Additional Activities

Mental Arithmetic

After the **Squares and Square Roots** lesson, as you work through the unit, you may ask students to approximate square roots without a calculator, and then see how close they got with the help of a calculator. For example, you might ask: "The square root of 90 is between which two whole numbers?" Do this often, as it is very important!

Equal Squares

A solid understanding of square roots helps lay the groundwork for solving quadratic equations by the *equal squares* method, which is spelled out in the Lab Gear materials, and in *Algebra: Themes, Tools, Concepts* 7.7 and 14.6. (*ATTC* is available free at www.MathEducationPage.org.)

Extensions

Geometric Puzzles

Tangrams and supertangrams are based on right isosceles triangles, and thus are intimately involved with the square root of 2. You can find tangram activities in *Geometry Labs*, and supertangram activities here: < http://www.mathedpage.org/puzzles/supertangrams/>

And More…

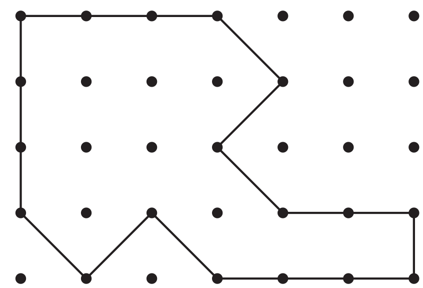
For more square root materials, see:

Geometry Labs 9.3, 9.4
Algebra: Themes, Tools, Concepts 9.3, 9.4, 9.5, 9.8, 9.9

LAB 8.4
Geoboard Area

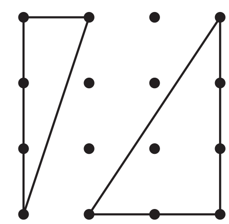
Name(s) ____________________

■ **Equipment:** Geoboard, dot paper



The area of the geoboard figure at right is 15.

1. Find other geoboard figures with area 15. The boundaries of the figures need not be horizontal or vertical. Find figures that are different from the ones your neighbors find. Record your solutions on dot paper.

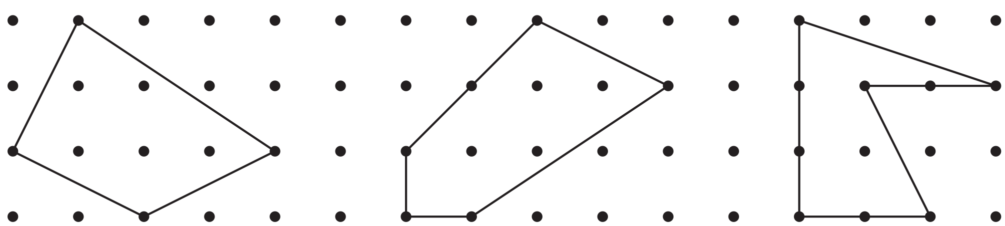


It is easiest to find areas of geoboard rectangles with horizontal and vertical sides. The next easiest figures are the "easy" right triangles, such as the two shown at right.

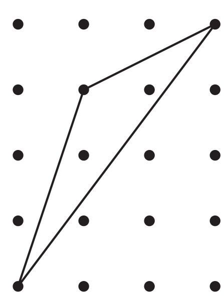
2. Find the areas of these triangles.

If you can find the area of easy right triangles, you can find the area of any geoboard figure!

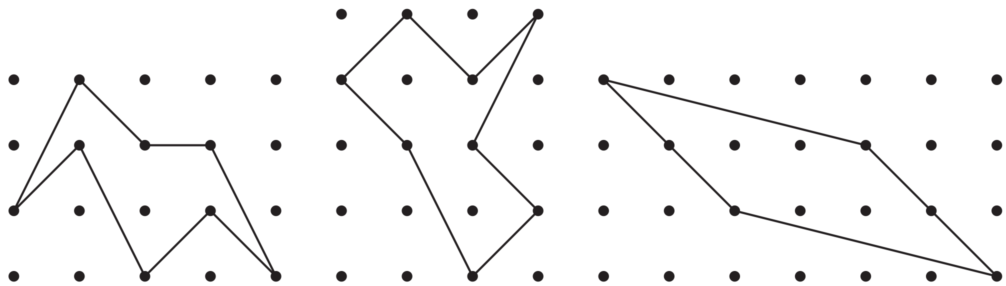
3. Find the areas of the figures below.



4. Find the area of the figure below. (It may be more difficult than Problem 3. **Hint:** Use subtraction.)



5. Find the areas of the figures below.



6. How many noncongruent geoboard triangles are there with area 8? Limit yourself to triangles that can be shown on an 11×11 geoboard and have a horizontal base. Record your findings on dot paper.

7. Puzzle: Find the geoboard figure with the smallest area in each of these categories.

a. Acute triangle

b. Obtuse triangle

c. Right triangle

d. Square

e. Rhombus (not square)

f. Rectangle (not square)

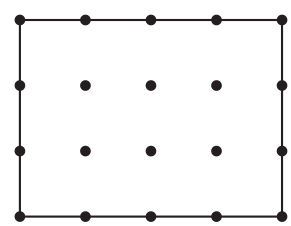
g. Kite

h. Trapezoid

i. Parallelogram

Discussion

A. A common mistake in finding geoboard areas is to overestimate the sides of rectangles by 1 (for example, thinking that the rectangle at right is 4×5). What might cause this mistake?



B. Explain, with illustrated examples, how the following operations may be used in finding the area of a geoboard figure: division by 2; addition; subtraction.

C. What happens to the area of a triangle if you keep its base constant and move the third vertex in a direction parallel to the base? Explain, using geoboard or dot paper figures.

D. Use geoboard figures to demonstrate the area formulas for various quadrilaterals.

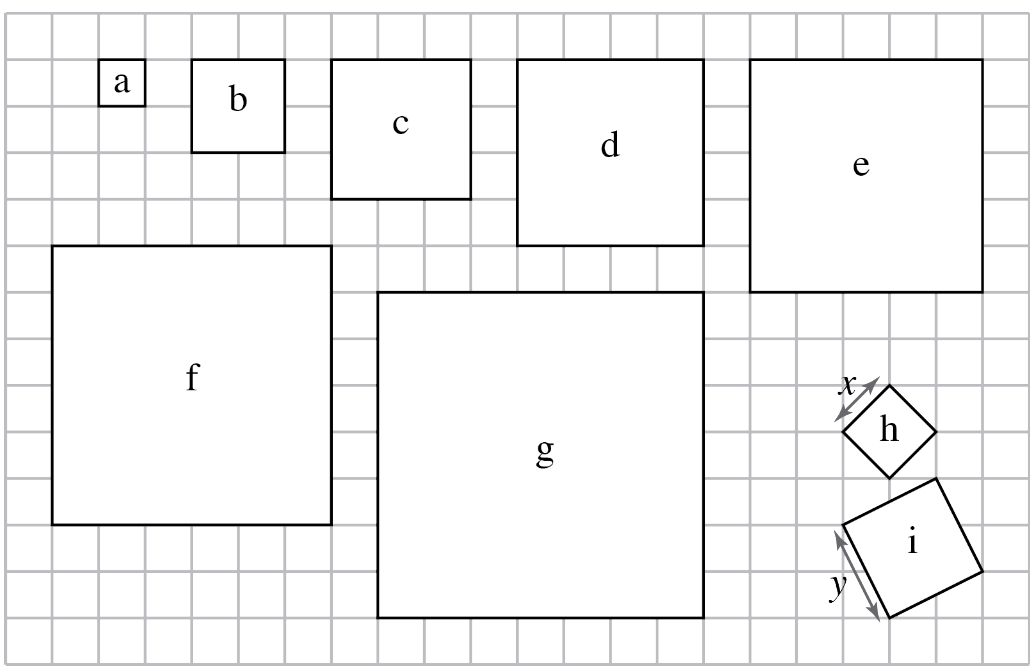
Squares and Square Roots

To find the area of a square, if you know its side, you multiply the side by itself: you *square* the side.

To find the side of a square, if you know its area, you take the *square root* of the area.

1. For each square, write three equations. Your equations should have the following forms:
   side · side = area, side^2 = area, and $\sqrt{\text{area}}$ = side

a.	b.	c.
d.	e.	f.
g.	h.	i.



2. Estimate *x* and *y* as decimal numbers (they are the sides of those squares).
   a. Without a calculator

   b. With a calculator

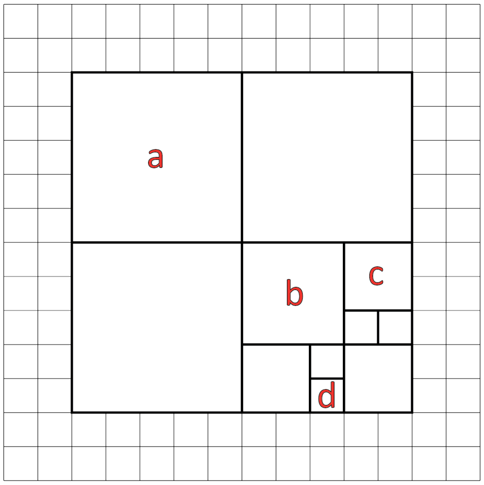
When the side of a square is a whole number, the area is called *a perfect square*. For example, 49 is a perfect square, because $\sqrt{49} = 7$, and 7 is a whole number. 2 is not a perfect square, because $\sqrt{2}$ = 1.414…, and 1.414… is not a whole number.

3. a. Name five perfect squares that are greater than 49.

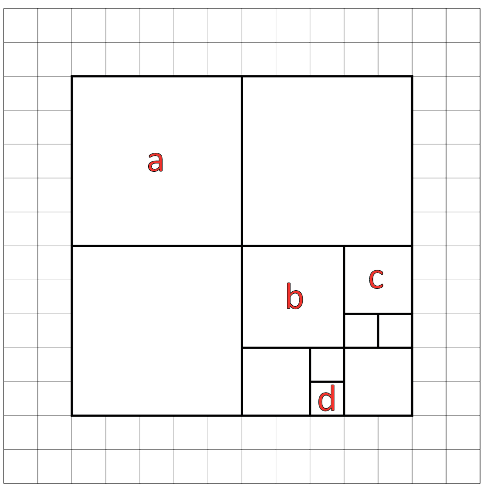
   b. Is 0 a perfect square?

   c. Is -49 a perfect square?

4. Fill out the tables with the areas and sides of squares a, b, c, d. If the answer is not a whole number, enter a decimal rounded to the nearest 1/100.

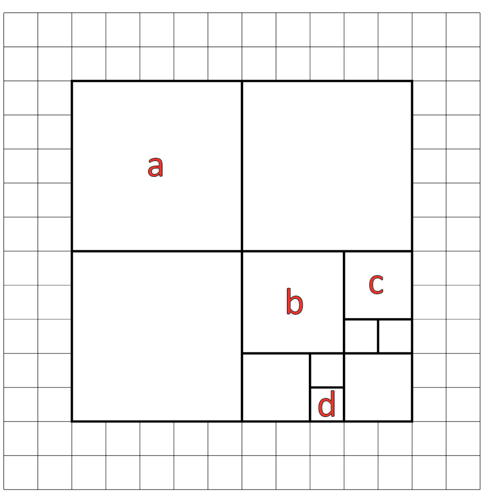


	Area	Side
Total	100	
a		5
b		
c		
d		



	Area	Side
Total	1	
a		
b		0.30
c		
d		

5. **Challenge:**



	Area	Side
Total	10	
a		
b		
c		
d	0.10	

LAB 8.5

Name(s) ______________________

Geoboard Squares

■ **Equipment:** Geoboard, dot paper

1. There are 33 different-size squares on an 11 × 11 geoboard. With the help of your neighbors, do the following.

a. Find all the squares.

b. Sketch each square on dot paper, indicating its area and the length of its side.

Discussion

A. How can you make sure that two sides of a geoboard square really form a right angle?

B. How can you organize your search so as to make sure you find all the squares?

C. Is it possible to find squares that have the same area, but different orientations?

D. In the figure at right, find the following in terms of *a* and *b*.

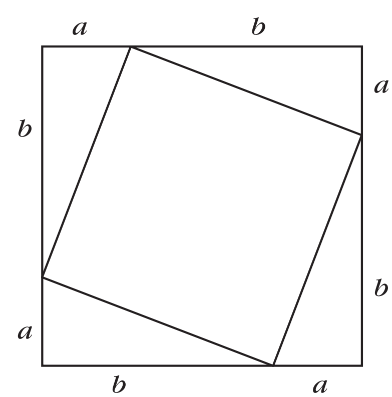
a. The side of the outside square

b. The area of the outside square

c. The area of each triangle

d. The area of the inside square

e. The side of the inside square



The Exponent ½

1. Write as a single power:
   a. $2^3 \cdot 2^4$

   b. $(4^3)^2$

   c. $\frac{3^5}{3^2}$

2. Find x.
   a. $2^5 \cdot 2^5 = 2^x$

   b. $2^3 \cdot 2^3 = x^6$

   c. $(2^4)^2 = 2^x$

3. Find x.
   a. $9^x \cdot 9^3 = 9^6$

   b. $9^x \cdot 9^x = 9^2$

   c. $9^x \cdot 9^x = 9^1$

   d. $B^x \cdot B^x = B^1$

4. Find x.
   a. $(9^x)^2 = 9^6$

   b. $(9^x)^2 = 9^1$

   c. $(B^x)^2 = B^6$

   d. $(B^x)^2 = B^1$

5. Problems 2-4 suggest a meaning for the exponent ½. Develop an explanation for what an exponent of ½ means.

6. Using this meaning of the exponent ½, find the following. If the answer is not a whole number, use a square root symbol.

   a. $16^{\frac{1}{2}}$

   b. $400^{\frac{1}{2}}$

   c. $25^{-\frac{1}{2}}$

   d. $2^{\frac{1}{2}}$