

## Dilation Teacher Notes

Teacher demonstrations using GeoGebra are essential at the beginning of this unit.

### 1. Dilation

The definition of a dilation is more complicated than the definitions of the isometries. Before explaining it, start by asking students to describe what they see in the figure, highlighting the most insightful observations.

In #2-3, students will find that all the measurements in the image have been multiplied by 3. The scaling factor is the constant of proportionality.

### 2. Scaling Factors

In this lesson, students are introduced to scaling factors that are between 0 and 1, and negative. Ask students how the figures differ from the ones in Lesson 1. Again, ask them to use a ruler to draw lines from the center of dilation to some vertices of the image and pre-image.

Do not expect students to solve #3 on their own. This needs to be a teacher-led class discussion, and you should refer to the previous two examples, and the one in Lesson 1 as needed. Better yet, use GeoGebra to demonstrate various cases. Ask these questions:

Which scaling factors lead to an image that is

- Smaller than the pre-image?
- Larger?
- Closer to the center of dilation?
- Further?
- Right side up?
- Upside down?

Note that in the first example, the center of dilation is inside the figure. That is not a problem.

### 3. Dilation Practice

Using a ruler is crucial. Let students choose their own pre-image polygons, but it is important that all the vertices be on the lines of the paper!

One likely difficulty is that some attempts will lead to images off the page. If that happens, students should reposition their center and/or pre-image and try again on another piece of paper.

### 4. Scaling Factors Summary

This lesson provides a needed review of the several different cases before launching into problem solving. Questions (a) through (d) can be answered by looking back at Lessons 1 and 2, and students' questions can be answered with the help of a GeoGebra projection.

You will probably need a class discussion of (e) through (g). If students find the questions challenging, remind them to go back to the definition of a dilation. What happens to the distance from the center of dilation in each case?

## 5. Dilation Puzzles

This brief activity can serve as a check on student understanding. It is not a full 55-minute lesson.

If students are stuck, you may need to remind them that the pre-image of a point and its image are collinear (on the same line) with the center of dilation.

| GeoGebra: this activity can be supported on the computer using interactive geometry software.

## 6. What is Preserved?

This lesson parallels a similar lesson about isometries.

Before starting, solicit predictions about what is preserved in a dilation, and what changes. For things that are expected to change, how will they change?

Students can work in groups of three or four, with each participant in charge of checking a different scaling factor, such as  $-2$ ,  $-\frac{1}{2}$ ,  $\frac{1}{2}$ , and  $2$ .

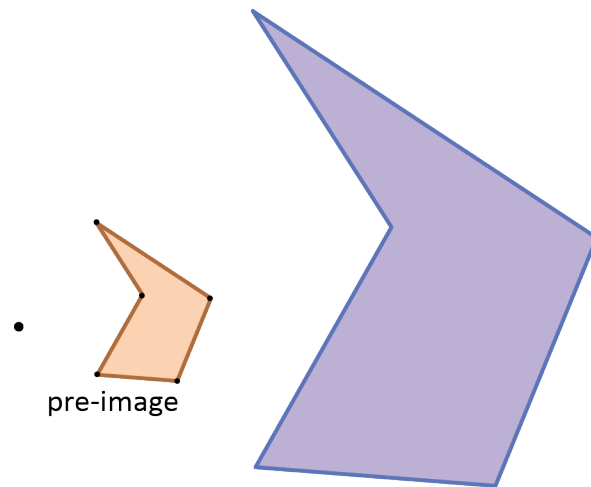
| GeoGebra: this activity can be supported on the computer using interactive geometry software.

## 7. Coordinates Puzzles

This lesson includes a review of isometries as well as questions about dilations.

## 1. Dilation

We will now learn about another type of transformation of the plane. It is called *dilation*, and is illustrated below.



A dilation has a *center*, and a *scaling factor*. In the example above, the center of dilation is the point marked in the figure to the left of the pre-image, and the scaling factor is 3.

To find the image of any point, draw a line through the point on the pre-image and the center of dilation. The image is on that line, at a distance from the center that is 3 times the distance from the center of dilation to the point on the pre-image.

1. In the figure above, the pre-image is a polygon.
  - a. Write “center” next to the center of dilation.
  - b. Measure the distance in millimeters from the center to the top left vertex of the pre-image. Then measure from the center to the corresponding vertex of the image.
  - c. Is the second distance you measured in part (b) 3 times as long as the first? It should be, because the scaling factor is 3.
  - d. Choose another vertex in the image, and use a ruler to draw a line from that vertex to the center. If you do it accurately, it should pass through the corresponding vertex in the pre-image.
  - e. Make measurements as you did in part b to check the scaling factor.

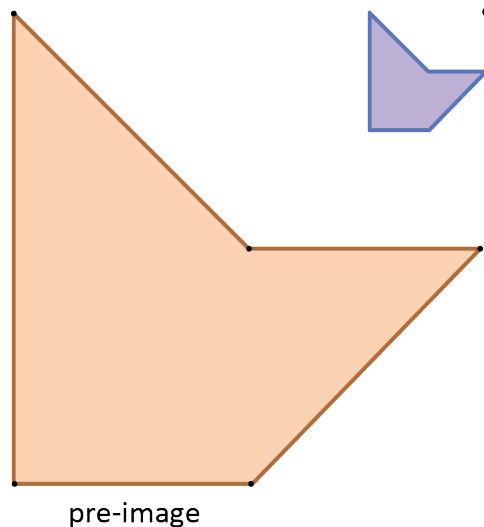
2. Measure the sides of the pre-image and image in millimeters, and enter your measurements in the table:

	<b>Pre-image</b>	<b>Image</b>
Side 1		
Side 2		
Side 3		
Side 4		
Side 5		

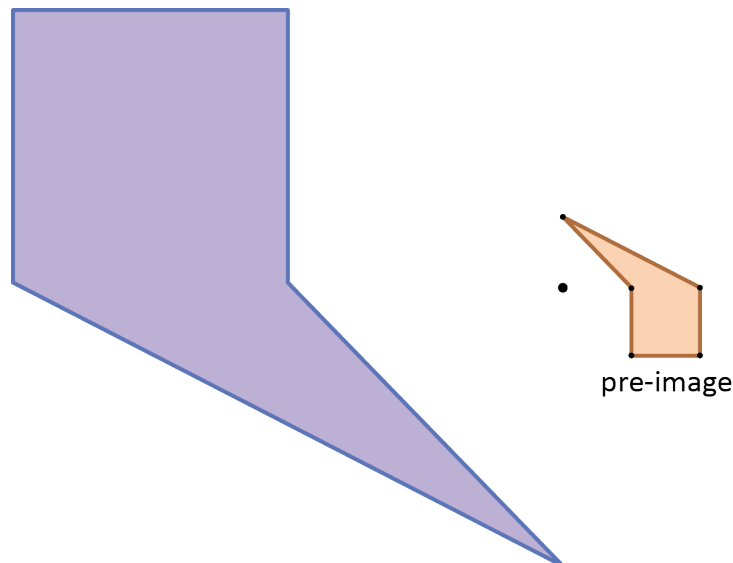
3. If your measurements are accurate, you should see a proportional relationship between the two columns. How is the constant of proportionality in the table related to the scaling factor between the figures?

## 2. Scaling Factors

In the previous lesson, the scaling factor was greater than 1. However, the scaling factor of a dilation can be between 0 and 1, and it can be negative, as you will see in the next examples.



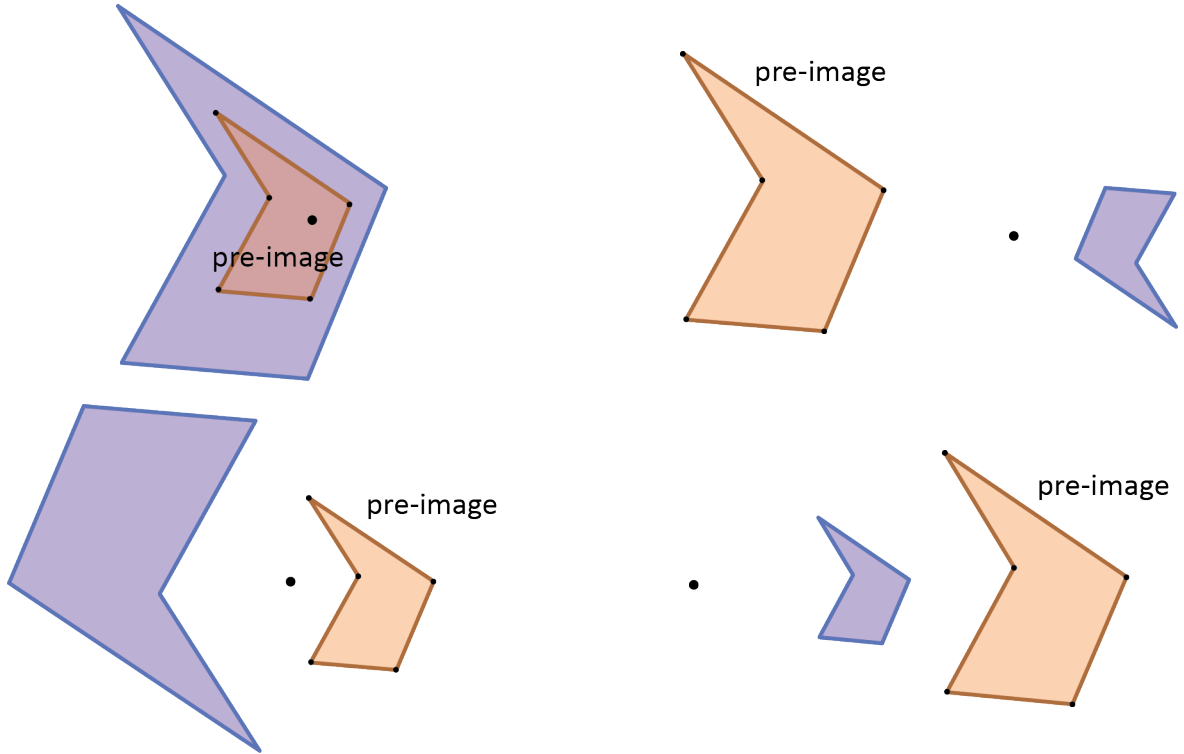
1. In the above figure, the scaling factor is  $\frac{1}{4}$ .
  - a. Verify the scaling factor by measuring in millimeters the distance from the center of dilation to a vertex of the pre-image, and then from the center to the corresponding vertex on the image.
  - b. If you measured accurately, the second measurement should be  $\frac{1}{4}$  of the first. Check whether it is.
  - c. Repeat (a) and (b) by measuring a *side* of the pre-image and the corresponding *side* on the image.



In the above figure, the scaling factor is negative. This tells us that to make the image, we need to go in the opposite direction from the center of dilation.

2. In fact, the scaling factor is  $-4$ . Use measurements to verify this.

Here are four more examples of dilations. The scaling factors are  $-2$ ,  $-\frac{1}{2}$ ,  $\frac{1}{2}$ , and  $2$ , but not in this order.



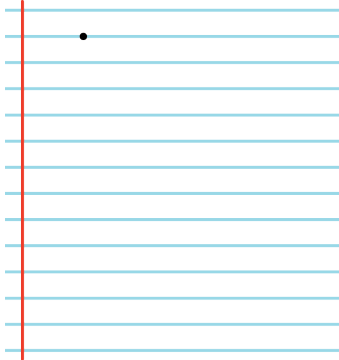
3. **Discussion:** Which scaling factor goes with which figure? Explain.

### 3. Dilation Practice

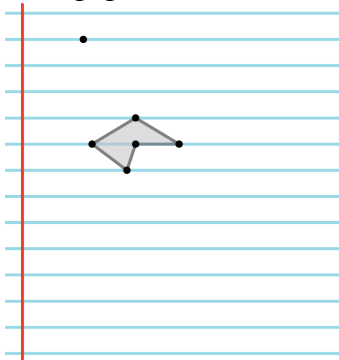
You will practice dilations on lined paper, with the help of a ruler.

#### 1. Scaling factor 2.

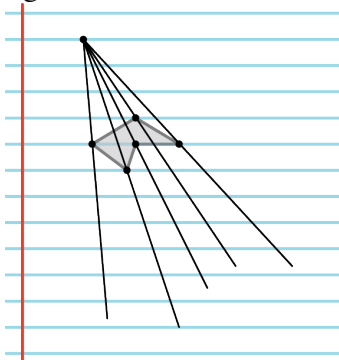
- a. Draw your center of dilation on a line near the top of the page.



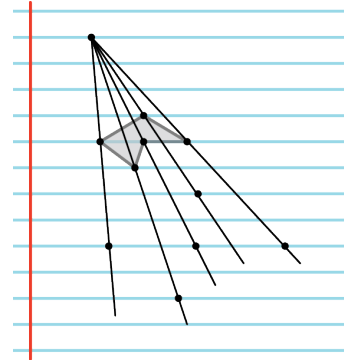
- b. A little further down, draw a polygon in such a way that all its vertices are on lines of the page.



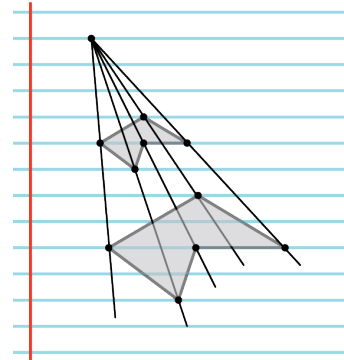
- c. Use your ruler to draw straight lines from the center to each vertex of your polygon, and extend each line more than twice as far down the page. You can do this by counting the number of lines crossed for the vertices of the pre-image, and doubling that number.



- d. Make dots for the vertices of the image polygon so that they are twice as far from the center as the originals.



- e. Connect the vertices to see the image.



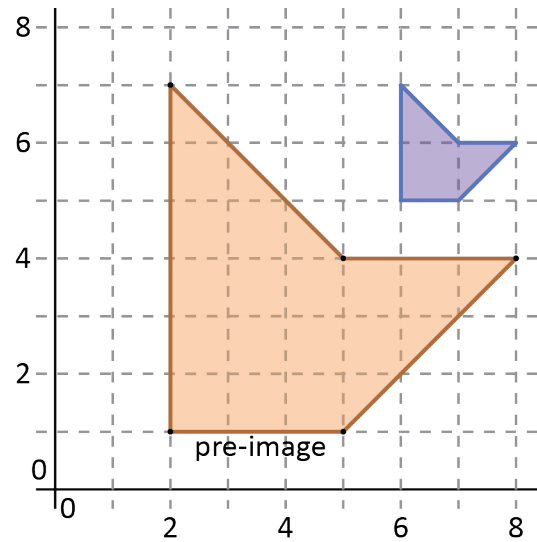
- Repeat #1 with scaling factor  $\frac{1}{2}$ . You will have to start with an original polygon further down the page.
- Repeat #1 with scaling factor  $-\frac{1}{2}$ . This time, your center should not be near the edge of the page.
- Repeat #1 with scaling factor  $-2$ .



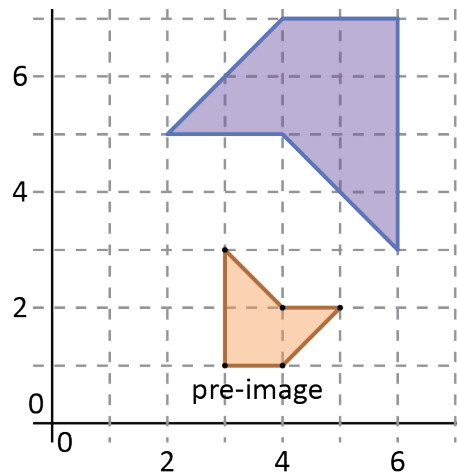


## 5. Dilation Puzzles

In each figure, one shape is a dilation of the other.



1. a. What is the scaling factor in the figure above?
- b. What are the coordinates of the center of dilation?



2. a. What is the scaling factor in this figure?
- b. What are the coordinates of the center of dilation?

## 6. What Is Preserved?

In this activity, you will compare a pre-image to its image after a dilation, and try to figure out what changes, and what stays the same.

Work with other students so as to be able to try different pre-images and different scaling factors.

Do your work on graph paper, and keep track of what you find out.

### Experiments

#### 1. Measurements:

- a. *Angles*: Draw a triangle, and create an image of the triangle using a dilation. Use patty paper to see if the angles in the image are equal to the angles in the pre-image.
  
- b. *Coordinates*: Choose three points that make a triangle and write down their coordinates. Create an image of each point using the same dilation for each. Did the coordinates change?
  
- c. *Length (distance)*: Draw a line segment and create an image of the segment using a dilation. Use patty paper to see if the length of the image is the same as the length of the pre-image segment. (You are checking whether the distance between the endpoints changed or stayed the same.)

#### 2. Relationships:

- a. *Collinear points*: Choose three points that lie on one straight line (not necessarily a graph paper line.) We say the three points are 'collinear'. Create images of the three points using the same dilation for each. Do the images of the points lie on one straight line?
  
- b. *Orientation*: Choose three points that make a triangle, and label them A, B, C, going around clockwise. Create images of your points using the same dilation for each, and label them A', B', and C'. If you go around A'B'C' in order, are you still going clockwise? (Clockwise and counterclockwise are how we describe the orientation of a triangle.)
  
- c. *Parallel lines*: Draw two parallel lines, and create images of the lines using the same dilation for each. Are the images of the two lines parallel?

## Conclusion

Based on your experiments, discuss with your group what changes and what stays the same after a dilation. Create a report on graph paper by writing down and illustrating your conclusions, making sure you all agree with each other.

Organize your report in three parts: (a) What changes? When things change, how do they change? (b) What stays the same? (c) Of the things that change, which depend on the scaling factor?

## 7. Coordinates Puzzles

The vertices of Triangle ABC have coordinates A: (1,1), B: (1,3), and C: (2,1). Use Triangle ABC as your pre-image in each problem 1-5.

1. After a transformation, the coordinates of the vertices of the image are:  
A': (-1,1), B': (-1,3), and C': (-2,1). Say as much as you can about this transformation.
  
2. After a different transformation, the coordinates of the vertices of the image are:  
A': (-1,-1), B': (-1,-3), and C': (-2,-1). Say as much as you can about this transformation.
  
3. After a different transformation, the coordinates of the vertices of the image are:  
A': (4,3), B': (4,5), and C': (5,3). Say as much as you can about this transformation.
  
4. **Challenge:** After a different transformation, the coordinates of the vertices of the image are: A': (5,1), B': (5,3), and C': (4,1). Say as much as you can about this transformation.

5. **Challenge:** After a different transformation, the coordinates of the vertices of the image are:  $A'$ :  $(-1,1)$ ,  $B'$ :  $(-3,1)$ , and  $C'$ :  $(-1,2)$ . Say as much as you can about this transformation.
  
6.
  - a. Draw the same triangle, the pre-image in Question 1, on graph paper.
  - b. Multiply all the coordinates by 3, and draw the resulting triangle.
  
  - c. Identify a dilation (center and scaling factor) to get the new triangle from the pre-image.
  
7. Guess what the coordinates of the image would be if you dilated the same pre-image using center  $(0,0)$  and scaling factor 2. Check your guess.
  
8. **Challenge:** Starting with your same pre-image, find a dilation that will have the same result as this sequence of transformations: reflect in the  $x$ -axis, and then reflect in the  $y$ -axis.