Chapter 3 Getting to Know the Lab Gear™

This chapter completes the introduction of the Lab Gear and of the basic Lab Gear techniques.

New Words and Concepts

This chapter introduces the use of the corner piece to structure the rectangle interpretation of multiplication and division. It is a crucial part of the Lab Gear which will be used repeatedly throughout the book.

Substituting is an important concept in applications of algebra, for example in the use of formulas in any science. It is also useful in checking the correctness of algebraic manipulations, and in bringing abstract work with variables down to the more concrete level of arithmetic. Doing the substitution with the blocks helps students avoid the "careless" arithmetic errors which are too frequent when doing this sort of manipulation.

The concept of **simplifying** expressions is introduced as well as the notion of **inequalities**.

Teaching Tips

In the long run, accuracy in substituting multi-digit values of the variable into complicated expressions is not a crucial skill. Programmable calculators and some computer software can automate the process, speed it up, and make it error-free. For example, in the Logo language to substitute 12.34 for x in the expression $5.67x^2 - 8.9x + 10.1112$ you can create the following procedure:

```
TO TRINOMIAL :X
OUTPUT 5.67 * :X * :X — 8.9 * :X + 10.1112
END
```

Once that procedure is in the Logo workspace, you can type

```
TRINOMIAL 12.34
```

to perform the desired substitution. In fact, you can rapidly carry out many substitutions. If your students have access to Logo, this is an important use of the language for them to learn.

Note that the shape of the corner piece resembles the symbol for long division, and that the dividend, divisor, and quotient appear in the appropriate positions.

Lesson Notes

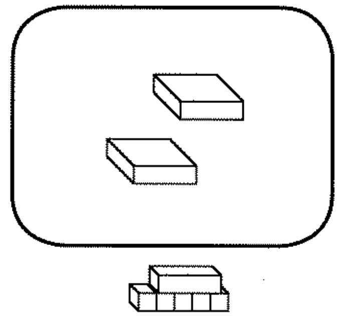
- **Lesson 1,** Substituting, page 30: You may want to demonstrate the substituting of a negative number for x on the overhead projector.
- **Lesson 2,** Using the corner piece, page 32: This is a straightforward example, using positive numbers. Use it as an introduction to the following exploration and lessons.
- **Lesson 3,** Multiplication, page 34: The rectangle model breaks down unexpectedly in some simple cases, such as problems 9 and 10, which cannot really be represented visually in a satisfactory manner—at least not with a rectangle. Hopefully, your students will be able to extrapolate from what they learned, and guess at the correct application of the distributive rule.
- **Lesson 4,** Division, page 35: Instead of dividend and divisor, the words numerator and denominator are used, for two reasons: they are more familiar to most students, and they underline the relationship between fractions and division, which cannot be stressed enough.
- **Lesson 5,** Simplifying, page 36: Solving inequalities provides the students with a reason for simplify the expressions. It is best to avoid simplifying "for its own sake" in unmotivated exercises.
- **Lesson 6,** Which is Greater?, page 39: Do not tell the students to add or subtract blocks from both sides of the inequality. They do not yet need this technique, and it is likely they will discover it for themselves in this lesson or in future "Which is Greater?" explorations. Next one: p.52.
- **Lesson 7,** Parentheses, page 42: Insist that students show both sides of the equation with blocks.
- **Lesson 8,** How Much More? How Many Times as Much?, page 43: This is an optional lesson.

Substituting

To solve these problems, you will need to **substitute** the given constants for the variables, cancel, and count. Be very careful with variables in the minus area, or upstairs.

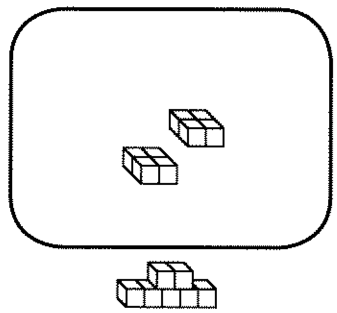
Look at this example.
These blocks show $-2x^2 - x + 5$.
Substitute these numbers for x.

a. 2
b. –3



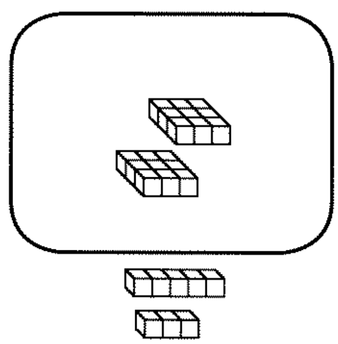
In part a, the substitution is straightforward. Put two 1-blocks in place of the x-block, and 2^2 (or four) 1-blocks in place of each x^2-block.

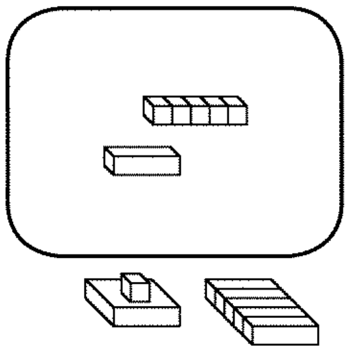
1. Write the final answer for part a.



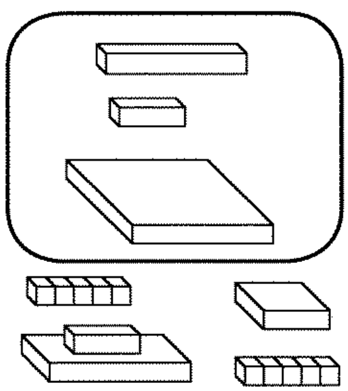
In part b, each x^2-block is replaced by $(-3)^2$, or nine 1-blocks.

2. Why was the x-block, which was upstairs, replaced by three 1-blocks *downstairs*?
3. Write the final answer for part b.



4. 

a. Write the expression shown by the figure above.
b. If $x = 2$, what is x^2? What value do the blocks show?
c. If $x = -3$, what is x^2? What value do the blocks show?
d. If $x = -5$, what value do the blocks show?

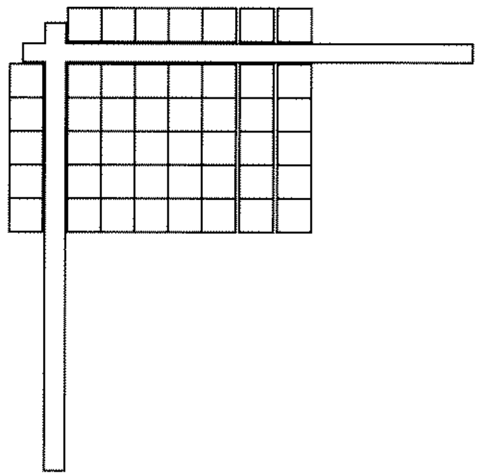
5. 

a. Write the expression shown by the figure above.
b. If $x = 1$ and $y = -2$, what is x^2? What is y^2?
What is xy? What value do the blocks show?
c. If $x = -3$ and $y = 2$, what value do the blocks show?
d. If $x = 5$ and $y = -5$ what value do the blocks show?

Using the Corner Piece

As you know, the result of any multiplication can be shown as a rectangle. This is because for a rectangle, **area = length · width**.

Arrange your corner piece and blocks to match this figure.

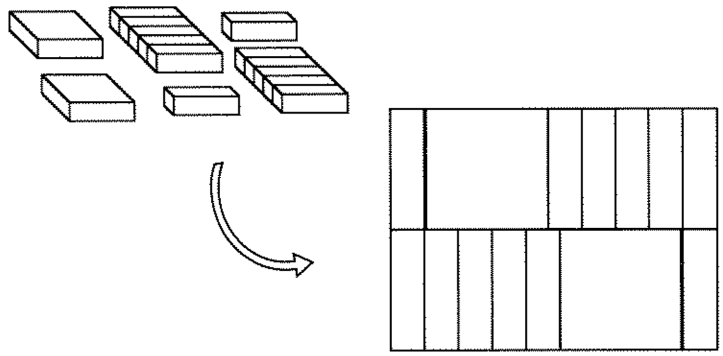


1. Write the multiplication equation that is shown by this figure.
2. Write a division equation that is shown by this figure.
   (area / length = width)
3. Write another division equation that is shown by this figure.
   (area / width = length)
4. Use the corner piece to set up as many different divisions as you can with numerator 12. Sketch each one. For each, write the division equation and the corresponding multiplication equation.
5. Explain why it is impossible to set up the division $\frac{12}{0}$ with the blocks.
6. Some algebra students believe that $\frac{12}{0} = 0$. Explain why they are wrong by discussing the multiplication that would correspond to this division.

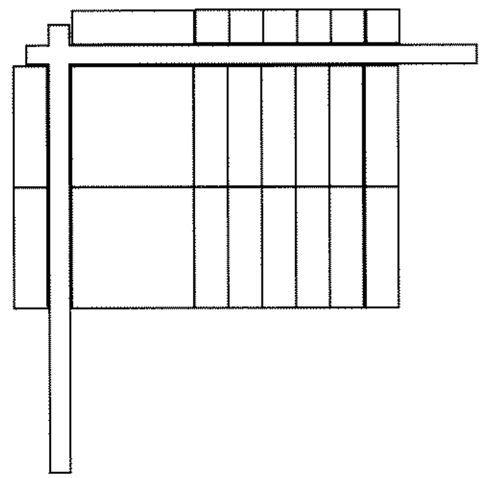
Exploration 1 Make a Rectangle

Sometimes a group of blocks can be arranged into a rectangle or square.

For example, you can rearrange the blocks $2x^2 + 12x$ into a rectangle like this.



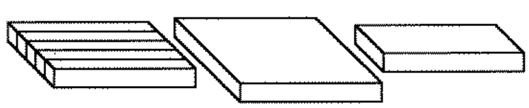
The length and width of this rectangle are $x + 6$ and $2x$, which can be seen better if you organize the blocks logically and use the corner piece, as shown. (Notice that you could also turn the rectangle so that the length and width are exchanged. This is considered to be the same rectangle.)



1. There is another rectangular arrangement of the same blocks which has different dimensions. Find it.

For each problem, arrange the given blocks into a rectangle. Sketch it (as seen from above) and write the length and width. Problem 4 has two solutions (find them both).

2. 

3. 

4. 

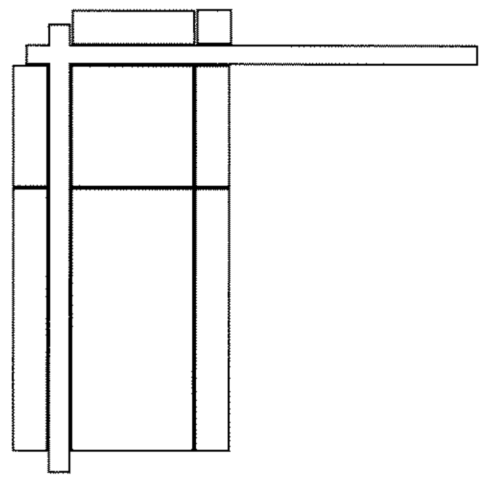
For each problem, try to guess the dimensions of a rectangle that could be made with the blocks. Then check your answer with the Lab Gear. One is impossible. Explain why.

5. $3x^2 + 9x$
6. $3xy + 2y + y^2$
7. $4x^2 + 9y$
8. $x^2 + 5x$

Multiplication

Arrange your corner piece and blocks to match this figure.

1. Write a multiplication and a division equation for the figure.



The multiplication equation one student suggested was $(x + y)(x + 1) = x^2 + x + xy + y$. To check whether this is correct, try substituting 3 for x and 2 for y on both sides of the equal sign. If you get a true statement, that will help convince you that the equality was valid.

$$(3 + 2)(3 + 1) \stackrel{?}{=} 3^2 + 3 + (3 \cdot 2) + 2$$
$$5 \cdot 4 \stackrel{?}{=} 9 + 3 + 6 + 2$$
$$20 = 20$$

2. Check again, by substituting other numbers for x and y.

Use the corner piece and your blocks to show these multiplications. Write the products.

3. $(x + 1)\,x$
4. $(x + 2)(x + 3)$
5. $(x + 5)(x + y)$
6. $2y\,(y + 1)$
7. $(y + 4)(y + 1)$
8. $(2x + 3)(x + y + 1)$

These multiplications cannot be modeled using the corner piece. Try to find the products another way.

9. $3\,(4x^2 + 6)$
10. $5\,(y^2 + 2y + 9)$

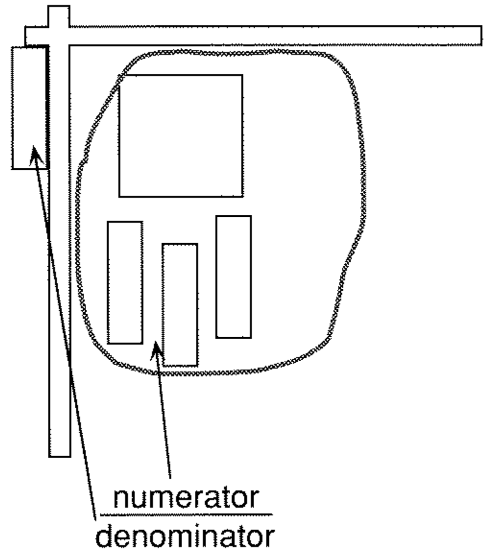
☑ Self-check

4. $x^2 + 5x + 6$
9. $12x^2 + 18$

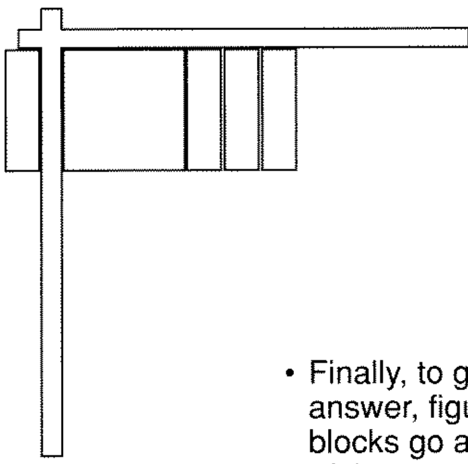
Division

Use your corner piece and blocks to follow this method for solving division problems.

- Put the denominator to the left of the corner piece.

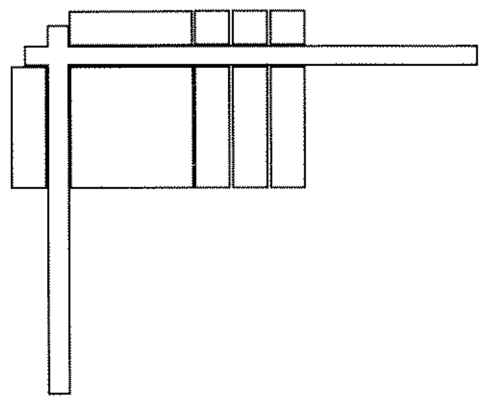


- Make a rectangle out of the numerator, and place it inside the corner piece so that one side of the rectangle matches the denominator.



- Finally, to get the answer, figure out what blocks go along the top of the corner piece.

1. Write the division equation shown by this figure.



Use this method to solve these division equations. Write the related multiplication equation. Not all are possible to solve with blocks.

2. $\frac{x^2 + 4x}{x} =$
3. $\frac{6x + 9}{3} =$
4. $\frac{x^2 + xy}{x} =$
5. $\frac{6x^2 + 3x}{3x} =$
6. $\frac{3y + 2}{5} =$
7. $\frac{4y^2 + 6y}{2y} =$
8. $\frac{2y^2 + y}{y} =$
9. $\frac{4y + 2x + 10}{2} =$

These division equations cannot be modeled clearly with the corner piece. Try to simplify them another way.

10. $\frac{6x^2 - 9}{3} =$
11. $\frac{4x^2 + 6x - 10}{2} =$

☑ Self-check

10. $2x^2 - 3$

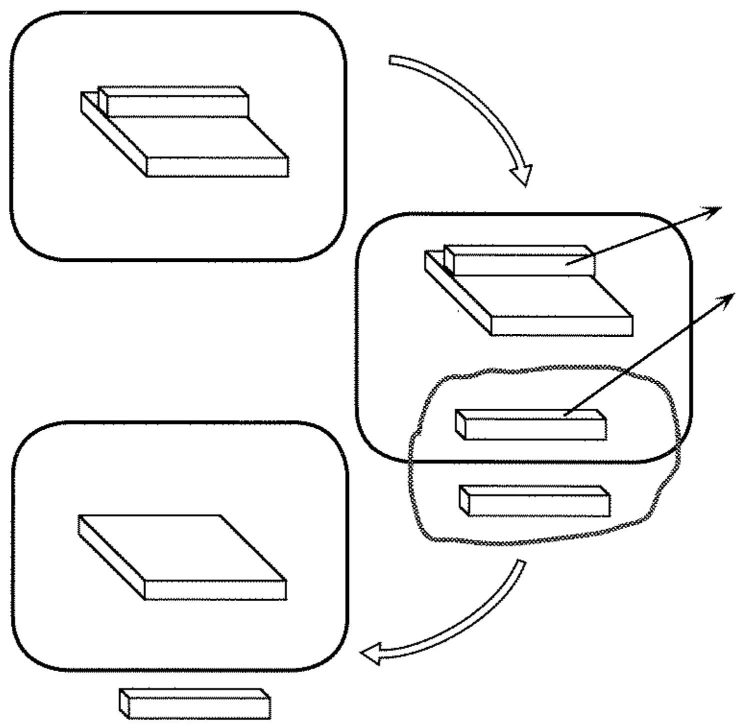
Simplifying

Here is a technique you will need to know. To simplify upstairs blocks in the minus area, you can use the adding zero trick.

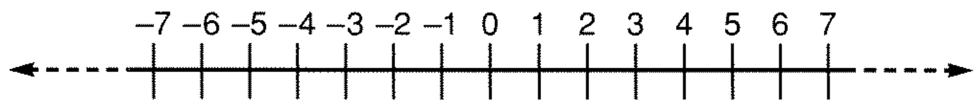
For example, this figure shows how to simplify $-(y^2 - y)$.

- Add the same quantity downstairs inside and outside of the minus area.
- Cancel matching upstairs and downstairs blocks.

The simplified form is $y - y^2$.



Which is Greater?

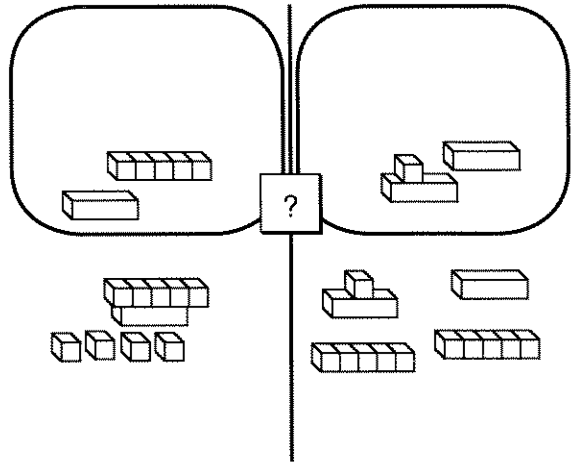


You can tell which of two numbers is greater by its position on the number line. The number that is greater is farther to the right. The number that is less is farther to the left. The symbol for *less than* is $<$. For example, $-5 < 3$, $0 < 7$, and $-6 < -2$. The symbol for *greater than* is $>$. For example, $6 > 3$, $0 > -2$, and $-5 > -9$.

1. Use the correct symbol.
   a. –5 **?** –7 b. –5 **?** –1

This workmat shows two expressions:
$x + 4 - 5 - (x + 5)$ and
$10 + 2x - 1 - (2x - 1)$.

Which is greater? The question mark shows that this is unknown.

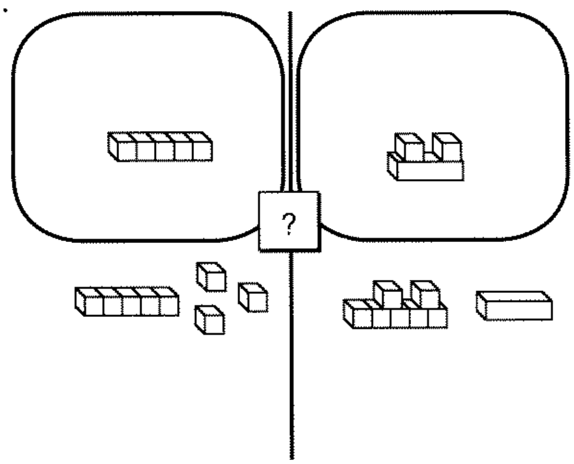


2. Mark the blocks that can be cancelled. Look at the blocks that remain. Write an expression for the blocks on the left side. Write an expression for the blocks on the right side. Which side is greater? Show your answer by writing the correct **inequality sign** between the two expressions.

For each problem, put out blocks to match the figure, and:

a. Write the two expressions.
b. Simplify both sides on the workmat.
c. Decide which side is greater or whether they are equal and write the correct sign between the expressions.

3.



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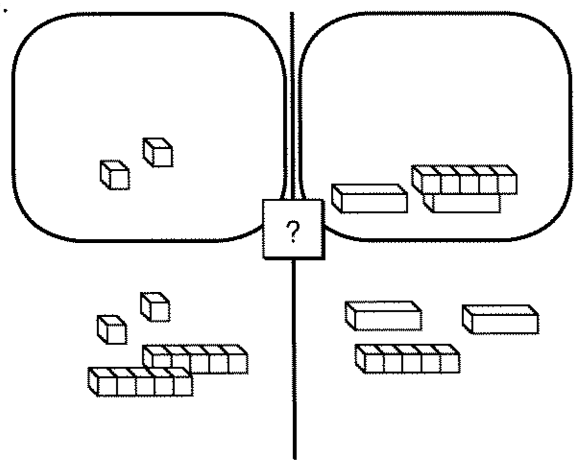

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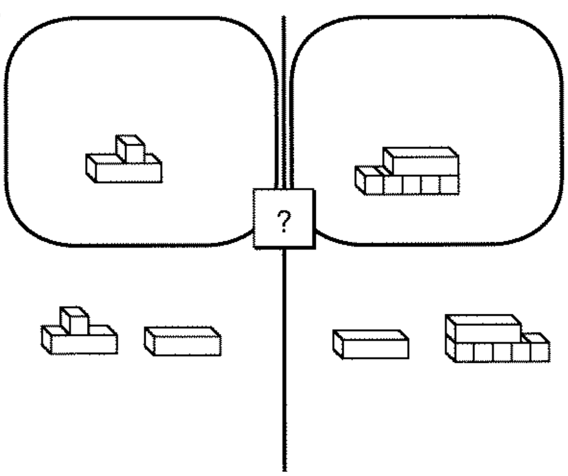

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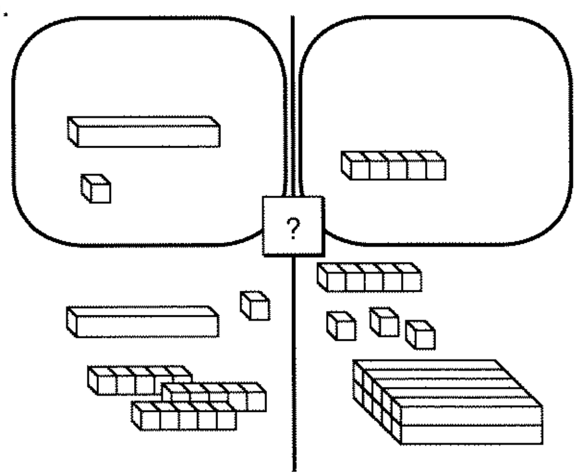
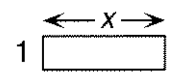


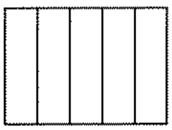
Exploration 2 Area and Perimeter

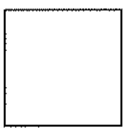
To determine the area and perimeter of the variable blocks, we will not use the actual measurements. Instead, we will consider their dimensions in terms of x and y.

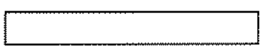
For example, this figure, the top of an x-block, is a 1 by x rectangle. So its area is $1 \cdot x = x$, and its perimeter is $x + 1 + x + 1$, which, by combining like terms, can be written $2x + 2$.



Find and write the area and perimeter of the following rectangles, which are the top faces of the remaining variable blocks. Be careful when collecting like terms.

1. 

2. 

3. 

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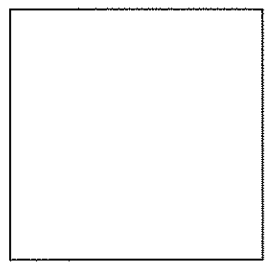
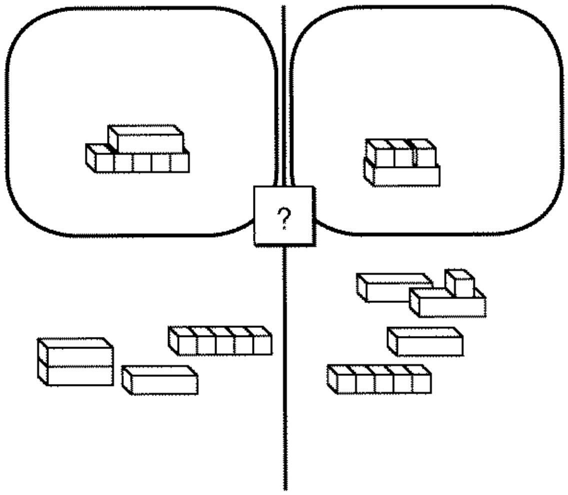
5. 

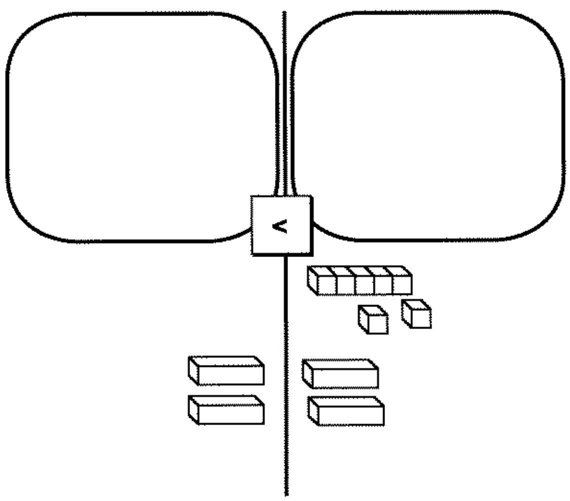
6. 

Which is Greater?

To compare $2x - x + 5 - (5 - x)$ with $5 + 3x - 1 - (x - 3)$, first show the two expressions with the Lab Gear.

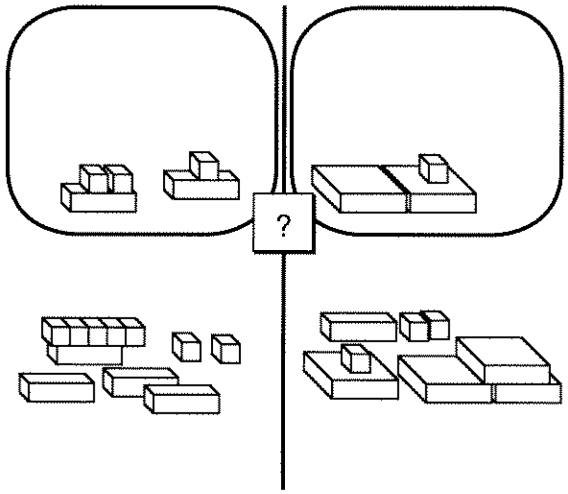


Simplify both sides and arrange the blocks in a logical manner to determine which side is greater. Both sides include $2x$, but the right side is greater as it also includes 7 more units.



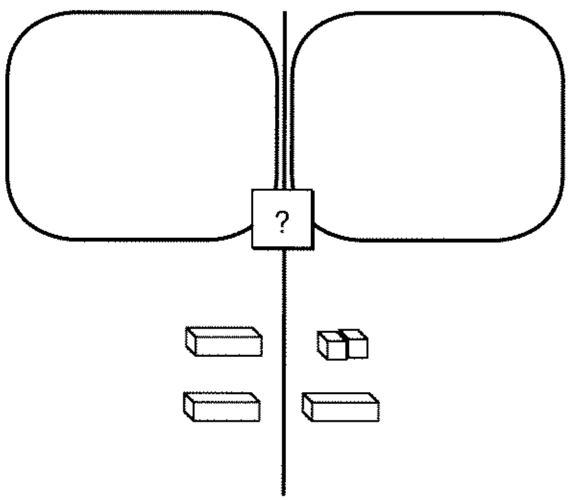
Now compare these expressions.

1. Write both expressions as they are shown in this figure.



2. Mark the blocks that can be cancelled. Now write the simplified expressions.

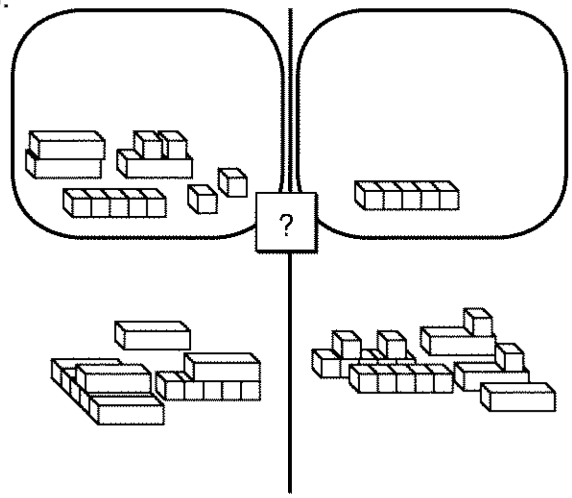
Your workmat should look like this.



In this case, it is impossible to tell which side is greater, because we do not know whether x is greater or less than 2.

For each of these problems, simplify using your blocks, and write the expressions in simplified form. Decide which side is greater, or whether they are equal, or whether it is impossible to tell. Write the correct symbol or “?”.

3.



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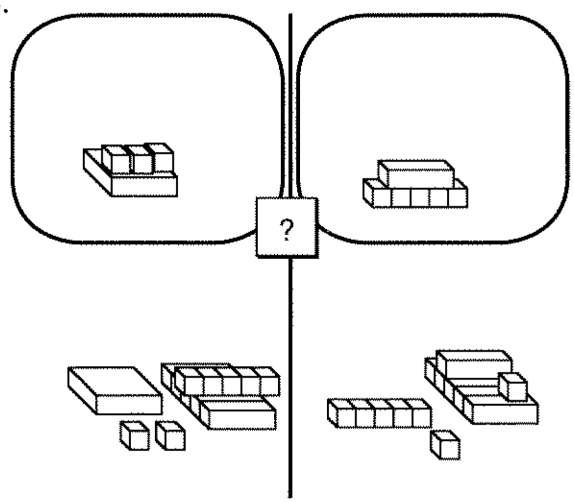

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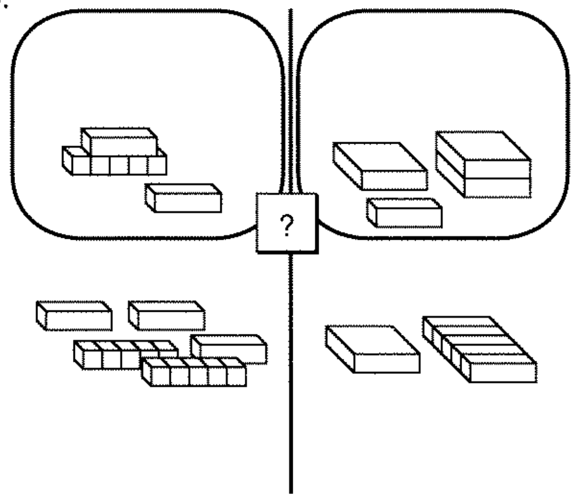

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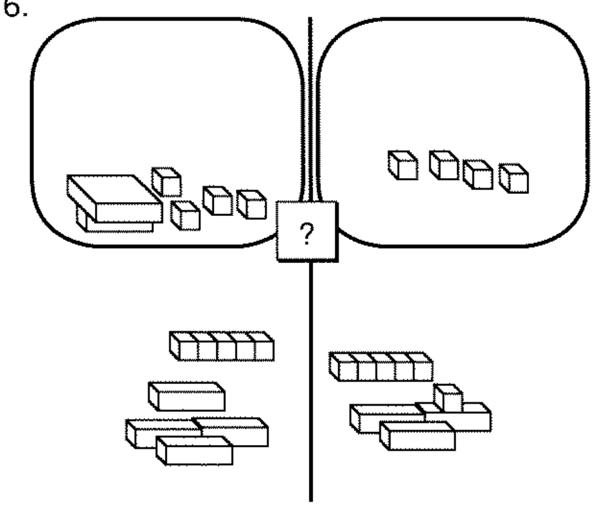


Parentheses

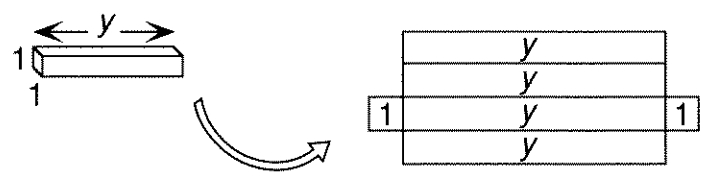
Use the Lab Gear to help you decide which of the expressions a, b, c, or d are equal to the expression on the left. Explain your answers.

1. $-(x+y)$	a. $-x+(-y)$	b. $-x-y$	c. $-x+y$	d. $y-x$
2. $-(x-y)$	a. $-x+y$	b. $-x-y$	c. $-x-(-y)$	d. $y-x$
3. $-(-x+y)$	a. $-x+y$	b. $-y-x$	c. $x-y$	d. $y-x$
4. $-(y-x)$	a. $x-y$	b. $-x+y$	c. $-y+x$	d. $-y-(-x)$

Exploration 3 Volume and Surface Area

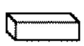
The surface area of the variable blocks can be figured out by thinking of their jackets.

For example, the y-block has surface area of $4y + 2$.



Its volume can be found by multiplying *length* times *width* times *height*. $1 \cdot 1 \cdot y = y$.

Find and write the volume and surface area of the remaining variable blocks.

1. 

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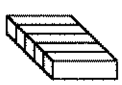
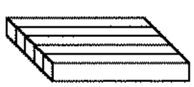
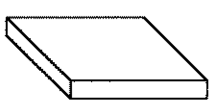
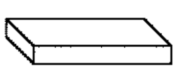
3. 

4. 

5. 

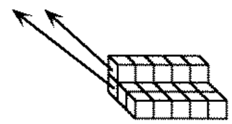
6. 

How Much More? How Many Times as Much?

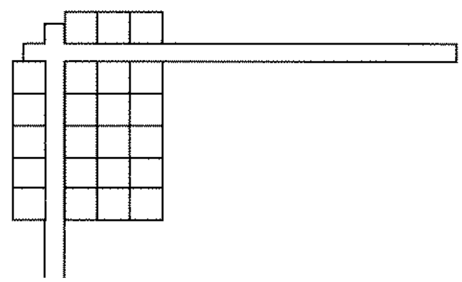
When comparing the size of two positive numbers, for example 5 and 15, you can ask two different questions.

- 15 is *how much more* than 5?
- 15 is *how many times greater* than 5?

Using the Lab Gear, the first question is answered like this. So, 15 is 10 more than 5.



The second question is answered like this. So, 15 is 3 times greater than 5.



Ask and answer both questions about these pairs of numbers. The Lab Gear may help you answer.

1. 25 and 5
2. 6 and 1
3. 4 and 2
4. 15 and 3
5. 42 and 7
6. 10 and 10
7. 9 and 8

Exploration 4 Always, Sometimes, or Never?

If a statement does not include any variables, it is either true or false. For example, $7 = 5 + 2$ is true, while $9 \cdot 7 = 54$ is false.

But if a statement contains one or more variables, there are three possibilities.

- Some statements are *always true*. For example, $x + 2 = 2 + x$. You can substitute any number for x, and you will get a true statement.
- Some statements are *never true*. For example, $x + 2 = 3 + x$. You can substitute any number for x, and you will get a false statement.
- Some statements are *sometimes true*. For example, $x^2 = 2x$. If you substitute a number for x, you will probably get a false statement. (Try 3, or 1, or –10.) But if you substitute 2 or 0 for x, you get a true statement.

Write A, S, or N for each of the following statements, depending on whether it is always, sometimes, or never true. Use the Lab Gear to help you decide.

1. $-(x-4) = -x+4$
2. $x + 4x + x = 6x$
3. $2 + x = 2x$
4. $y(y+2) = y^2 + 2$
5. $2x + 5 = 2x + 1$
6. $\dfrac{x^2 + 2x}{x} = x + 2$
7. $y^2 - y - 1 = y^2 - y - 5$
8. $x + 15 = 2x$