

Letters and Postcards

Alice, a mathematician with many friends, is visiting San Diego. She is thinking of sending a bunch of postcards, and proceeds to make a list of the possible recipients. As she makes the list, she realizes that she may end up deciding to write letters to some of her friends, instead of postcards. In addition to 14 friends to whom she absolutely must write, there are 4 friends to whom she may write. She already has all her supplies, plus \$4.00 to buy stamps. Postcard stamps cost 20 cents. Letter stamps cost 32 cents. “Hmm, she says to herself, this would make a good word problem!”

1. Find several combinations of postcards and letters that satisfy the conditions outlined above. (For example, Alice may send 10 postcards and 5 letters, at a cost of \$3.60 in stamps.)
2. Check whether a neighbor's combinations do satisfy the constraints: is Alice sending an acceptable total number of messages? Can she afford the stamps?
3. Say that x = the number of postcards, and y = the number of letters. Graph all possible combinations of x and y given the above constraints by marking each possible (x,y) combination with a dot. (Use axes from 0 to 20 for each of x and y , with one graph paper unit = 1 card or letter. If you do this correctly, you should have 37 dots.)

The area where the dots are is called the *feasible region*.

4. Write inequalities involving x and y that express that Alice will:
 - a. send 14 messages or more (answer: $x+y \geq 14$)
 - b. send 18 messages or fewer
 - c. spend \$4.00 or less on stamps
 - d. send a non-negative number of letters
 - e. send a non-negative number of postcards
5. Graph five lines, one for each of the inequalities you found in the previous problem. Label each line with its equation. (Example: $x+y=14$.)
6. Describe where on your graph you would find an (x,y) pair that corresponds to:
 - a. too few messages sent
 - b. too many messages sent
 - c. too much spent on stamps
 - d. exactly \$4.00 spent on stamps
7. The total cost of a postcard is 70 cents (50 cents for the card, 20 cents for the stamp.) The total cost of a letter is 35 cents (32 cents for the stamp, .5 cent for the paper, 2.5 cents for the envelope.) Ignoring any other costs, what is the cheapest combination for Alice? The most expensive combination? Label them on the graph.
8. It takes Alice 3 minutes to write a postcard and 15 minutes to write a letter. Which is the least time-consuming combination? The most? Label them on the graph.