Connect The Dots!

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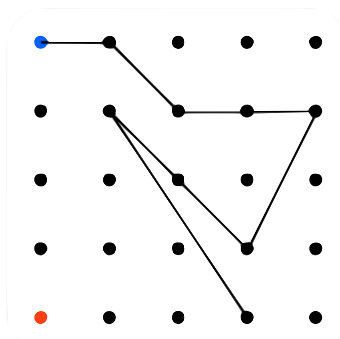
I mostly don't know the answers to these problems. Some are unsolved, or related to unsolved problems, and were suggested by Gord Hamilton (mathpickle.com).

n by *n* Lattices

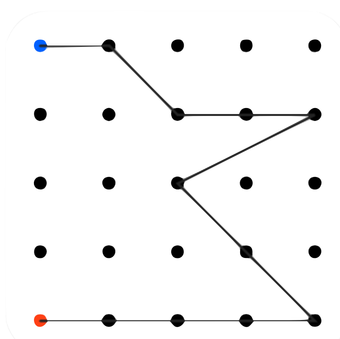
1. What is the greatest number of lattice points you can choose on an n by n lattice so that **no three are collinear**?

2. What is the greatest number of **unit square diagonals** one can draw on an n by n lattice so that they are completely disjoint (they do not intersect or share an endpoint)?

3. **Create a path** from the top left to the bottom left of an n by n lattice so that each step is a line segment joining lattice points. Each step must be longer than the previous step. The path cannot cross itself. For example, this is not a successful trip on a 5 by 5 lattice, as the path did not reach the destination:



(Taking the last step to the exit would violate the rules, as it would be shorter than the previous step.) On the other hand, this is a successful trip in six steps:



The challenge is to get to the exit after taking the **greatest possible number of steps**. (For example, on a 5 by 5 lattice, the optimal number is 9.)

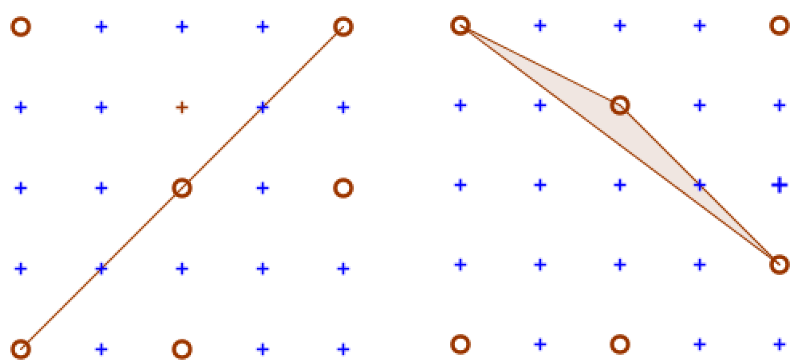
11 by 11 Lattice

11 by 11 is a standard size for geoboards.

4. Find all **isosceles geoboard triangles**, excluding right triangles and any whose base is parallel to or at a 45° angle from the edge of the board.

5. Find all geoboard **triangles with area 15**, such that no side is parallel to the edge of the board. (They don't need to be isosceles.)

6. For a given set of lattice points, any three determine a triangle (possibly a "flat" triangle with area 0.) Among those triangles, call a triangle with least area a *Heilbronn triangle*. Choose k lattice points to **maximize the area of the Heilbronn triangle**. For example, for $k = 6$, these arrangements are not optimal, as the Heilbronn triangles have area respectively 0 and 1. It is possible to arrange six dots and get a Heilbronn triangle with area > 1. Find the optimal arrangement for k = 4, 5, 6, and 7 on an 11 by 11 lattice.



More

7. **Pick's formula.** Let us call simple polygons (no crossings, no holes) with vertices on lattice points *geoboard polygons*. Pick's formula relates the area of a geoboard polygon (A) to the number of interior lattice points (I) and boundary lattice points (B). Find Pick's formula. (A proof that the formula is correct in all cases can be found on my Web site.)

Most of these problems and puzzles can be used in the classroom or with a math club.

For more directly curricular uses of the geoboard (area, distance, slope, Pythagoras, square roots) see:
www.mathed.page/geoboard