LESSON 13.4

Rectangular Pens: Constant Area

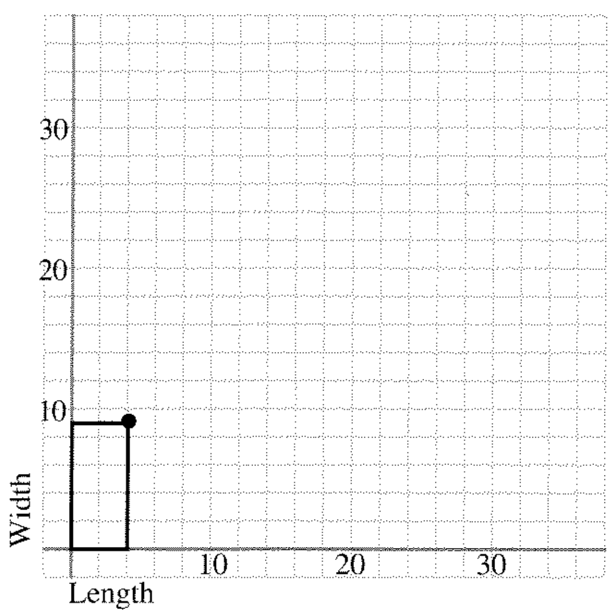
You will need:

- graph paper
- a graphing calculator (optional)

1. **Exploration** You bought 45 square feet of artificial turf for the floor of Stripe's backyard. You can cut it up any way you like, but you want to use all of it. Since you're almost broke (artificial turf is expensive) you would like to spend as little money as possible on fencing. What's the least amount of fencing you could buy and still make a rectangular pen that surrounded the artificial turf on all four sides? Find out by trying various dimensions for the pen.

WIDTH AS A FUNCTION OF LENGTH

Suppose you want to make a rectangular pen having area 36.



2. a. On graph paper draw a pair of axes and show five rectangular pens that would have an area of 36 square feet. The lower left corner should be at the origin. An example is shown in the figure.

b. Mark with a • the upper right corner of each rectangle you drew. Then write in the coordinates of each of these points.

c. Connect the •s. Do they lie in a straight line or on a curve? Describe any patterns you notice.

3. Make a table showing some of the coordinates on your graph. Look for a pattern in your coordinates and make three more entries in the table.

4. Write an algebraic equation that expresses the width as a function of the length.

5.

a. Would it be possible to have a pen having length greater than 30? 32? 36? Explain your answers, giving examples.

b. Explain why your graph will never touch the x-axis or the y-axis.

c. If you increase the length by one foot, does the width increase or decrease? Does it change by the same amount each time? Explain.

PERIMETER LINES

In the previous section you probably noticed that the perimeter of the rectangles changed even though the area remained constant. In this section you will investigate how the perimeter varies as a function of length if you keep the area constant.

6. Write the perimeter of the corresponding rectangle next to each • you marked on the graph. Look for patterns.

Your graph should show pairs of points that correspond to the same perimeter. For example, (3, 12) and (12, 3) both correspond to the perimeter 30.

7. Connect (3, 12) and (12, 3) to each other by a straight line. Extend it to its intercepts. Interpret the intercepts in terms of this problem.

8. On your graph find two points that both correspond to a perimeter of 26. Repeat problem 7 for these points. Then find other pairs of points that both correspond to the same perimeter and repeat problem 7 for each of these pairs. What patterns do you see?

9. Use the graph to estimate the dimensions of a rectangle having area 36 and perimeter 36.

PERIMETER AS A FUNCTION OF LENGTH

10. Make a graph of perimeter as a function of length. Show length on the x-axis and perimeter on the y-axis. Connect the points on your graph with a smooth curve. Describe the shape of the curve.

11. Label the lowest point on your graph with its coordinates. Interpret these two numbers in terms of the problem.

Note: The graph is *not* a parabola, and its lowest point is *not* called a *vertex*.

12. Explain why your graph will never touch the x-axis or y-axis.

13. If you increase the length by one foot, what happens to the perimeter? Can you tell whether it will increase or decrease? Does it increase or decrease by the same amount each time? Explain.

14. **Summary**
    a. For a fixed area of 36 square feet, explain in words how you would find the perimeter of the rectangular pen if you were given the length.
    b. If the area of a rectangular pen is 36 and its length is L, write an algebraic expression for its perimeter.
    c. If you had to enclose a rectangular area of 36 square feet and wanted to use the least amount of fencing, what would the length, width, and perimeter be? Explain.

Generalizations

15. If the area of a rectangular pen is A and its length is L,
    a. write an algebraic expression for its width in terms of A and L;
    b. write an algebraic expression for its perimeter in terms of A and L.

16. Explain how to find the length that gives the minimum perimeter. Write an algebraic expression for it in terms of A only.

NUMBER PUZZLES

17. Find two numbers x and y whose product is 75 and whose sum is 20. Explain your method.

18. Graph the equations $xy = 75$ and $x + y = 20$ on the same pair of axes. Find their point of intersection. How is this point related to your answer to problem 17?

19. Find two numbers whose product is 75 and whose sum is 23.75.

20. If two numbers have a product of 75, what is the smallest value their sum could take? What is the largest? Explain.