Essential Ideas

FIRST CLASS STAMPS

This table shows the costs in cents of first-class stamps over the past sixty years. The dates indicate the year when there was an increase in the first-class rate.

Year	Cost
1932	3
1958	4
1963	5
1968	6
1971	8
1974	10

Year	Cost
1975	13
1978	18
1981	20
1985	22
1988	25
1991	29

Interpolation is not relevant since all the data are known within the given period. However extrapolation may be possible.

1. Graph the data as a step function. For example, the cost was 3 cents from 1932 to 1957.
2. In 1985 Barbara wanted to use the average cost increase in the period 1932-1985 to predict the cost of stamps in 1991.
   a. What was the average yearly increase?
   b. Based on this, what cost did she predict for 1991?
3. In 1985 Sue used a computer to find the average percent increase over the 53-year period. The computer indicated that on the average, the cost went up by 3.8% a year. Based on this, what cost did she predict for 1991?

In 1991 they used the same methods to find the average increases over the 59-year period. Barbara found an average increase of 0.44 cents a year, and Sue found an average percent increase of 3.9% a year.

4. Make a prediction for the cost of stamps in the year 1999 and 2032. Explain.

THE MILE RUN

Year	Time
1868	4:29.0
1868	4:28.8
1874	4:26.0
1875	4:24.5
1880	4:23.2
1882	4:21.4
1882	4:19.4
1884	4:18.4
1894	4:18.2
1895	4:17.0
1911	4:15.6
1911	4:15.4
1913	4:14.6
1915	4:12.6
1923	4:10.4
1931	4:09.2
1933	4:07.6
1934	4:06.8
1937	4:06.4
1942	4:06.2

Year	Time
1942	4:04.6
1943	4:02.6
1944	4:01.6
1945	4:01.4
1954	3:59.4
1954	3:58.0
1957	3:57.2
1958	3:54.5
1962	3:54.4
1964	3:54.1
1965	3:53.6
1966	3:51.3
1967	3:51.1
1975	3:51.0
1975	3:49.4
1979	3:49.0
1980	3:48.8
1981	3:48.53
1981	3:48.40
1981	3:47.33

5. The table shows the world record for the mile run from 1868 to 1981. Plot the time *in seconds* as a function of year.

6. Use the median-median line method to fit a line.

7. What is the equation of your fitted line?

8. Richard Webster of Great Britain ran the mile in 4:36.5 in 1865. How does this compare with the time for 1865 predicted by your fitted line?

9. Steve Cram of Great Britain ran one mile in 3:46.31 in 1985. How does this compare with the time predicted by your fitted line?

10. a. According to your model, when would the mile be run in 0 seconds?
    b. For how many more years do you think your fitted line will be a good predictor of the time?

WIRES

When a metal wire changes temperature, it expands or contracts, according to the equation

$$L = L_0(1 + kT),$$

where L is the length of the wire, L_0 is its length at 0°C, T is the temperature, and k depends on the metal. For copper, $k = 1.8(10^{-5})$.

11. A copper wire is 100.05 meters long at 40°C. If it is cooled to -10°C, how much will it shrink? (Hint: First find its length at 0°C.)

12. Two poles are 100 meters apart. They are connected by a 100.05-meter copper wire in the summer, when the temperature is 40°C. In the winter the temperature drops to -10°C.
    a. Explain why the wire breaks.
    b. 💡 How long should the wire be so as not to break in the winter?

A nickel-iron alloy is created. Measurements are made in a lab on a wire made of the alloy. It is found that a wire that is 10 meters long at 0°C expands by one half a millimeter at 100°C. The alloy is called *Invar.*

13. Find the value of k for Invar.

14. 💡 Would an Invar wire that measures 100.01 meters at 40° C work to connect the poles in problem 12? Explain.

EQUATIONS FROM DATA

a.

x	y
0.4	15
0.6	10
0.8	7.5
1	6

b.

x	y
0.4	0.667
0.6	1.00
0.8	1.33
1	1.67

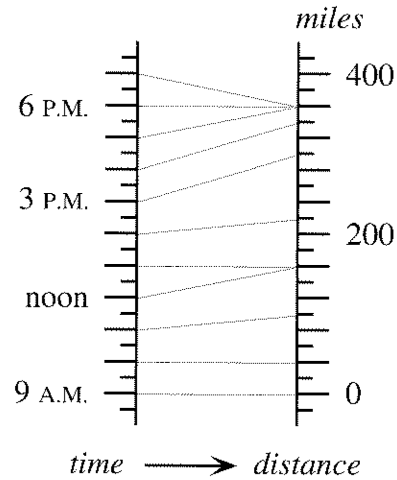
c.

x	y
0.4	-4.4
0.6	-2.6
0.8	-0.8
1	1

15. Find an equation for each table. Hint: One is a direct variation, one an inverse variation, and one a linear function.

THE CAR TRIP AND THE BICYCLE TRIP

Reread **Thinking/Writing 2.B** (Chapter 2). The function diagram is shown below.



Car Trip

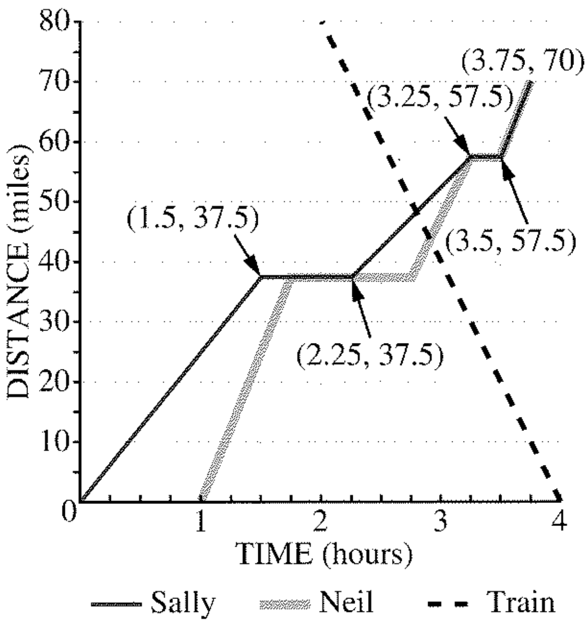
16. Make a Cartesian graph for the car trip, as best you can from the information given.

17. What is the car's average speed,

a. if you include the time the car was stopped in the middle of the day?

b. if you include only the driving time?

Reread **Thinking/Writing 4.A** (Chapter 4). The graph is shown below.



18. Repeat problem 17 for the van Neil drove.

19. If Neil were to make the return trip in the same length of time, but traveling at a constant speed and never stopping, what would be his speed?

20. a. Write an equation for Sally's graph during the leg of the trip when she and the train passed each other.

b. Solve the system of equations consisting of the equations representing Sally's and the train's motion.

c. Interpret the point of intersection.

ASTHMA

For her asthma Lynne takes 360 mg of the drug *theophylline* twice a day. After 12 hours, 60% of the drug has been eliminated from her body.

21. Assume Lynne has x_a mg of the drug in her body immediately after taking the dose. Explain why $y_a = 0.4y_a = 0.4x_a + 360$ is the recurrence equation that says how much will be in her body immediately after taking the next dose.

22. Assume she has x_b mg of the drug in her body immediately before taking the dose. Explain why $y_b = 0.4(x_b + 360)$ is the recurrence equation that says how much will be in her body immediately before taking the next dose.

The amount of theophylline in Lynne's body is constantly changing, but the lowest amount (right before taking the drug) and the highest amount (right after) eventually approach a stable level.

23. Find that level, using tables, function diagrams, or equations. What is the level before taking the dose? What is it after?