

The Bureau of the Census conducts a census every ten years, as required by the U.S. Constitution. Census results are now used for many purposes, but their original purpose was primarily to determine how many seats each state would be allocated in the House of Representatives. As population patterns change, these seats are divided up differently among the states. Here are some census results from 1890 through 1990.

Census Table

| Year | Population | \# increase | \% increase |
| :---: | :---: | :---: | :---: |
| 1890 | $62,979,766$ | $12,790,557$ | 25.5 |
| 1900 | $76,212,168$ | $13,232,402$ | 21.0 |
| 1910 | $92,228,496$ | $16,016,328$ | 21.0 |
| 1920 | $106,021,537$ | $13,793,041$ | 15.0 |
| 1930 | $123,202,624$ | $17,181,087$ | 16.2 |
| 1940 | $132,164,569$ | $8,961,945$ | 7.3 |
| 1950 | $151,325,798$ | $19,161,229$ | 14.5 |
| 1960 | $179,323,175$ | $27,997,377$ | 18.5 |
| 1970 | $203,302,031$ | $23,978,856$ | 13.4 |
| 1980 | $226,545,805$ | $23,243,774$ | 11.4 |
| 1990 | $248,709,873$ | $22,164,068$ | 9.8 |

## WVVMTM11

1. Use the information given to estimate the 1880 population.
2. Write a paragraph describing anything interesting you see in the data. What factors affect population growth? Can you think of historical events that might be associated with periods of low or high growth rates?
3. Over what ten-year period was
a. the number increase the smallest? The largest?
b. the percent increase the smallest? The largest?
4. From 1890 to 1990 , what was the overall
a. number increase?
b. percent increase?
5. Using a large piece of graph paper, make a graph of the population as a function of time. You will add to this graph when you do other problems in this lesson. Choose the scale carefully.
6. Approximately when did the population reach its halfway point between 1890 and 1990? Explain how you calculated this, and show the point on your graph. Is the halfway point in population before or after the halfway point in years?

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If you had lived in 1890 and wanted to predict the population of the United States in 1900 and 1990, you might have assumed that the increase for every future ten-year period would be the same as it had been in the ten-year period from 1880 to 1890 . (We will call 18801890 the base period.)
7. For each assumption below, make a table showing what the population would have been in each decade.
a. The number increase in each decade was the same as in the base period.
b. The percent increase in each decade was the same as in the base period.
8. In this problem, add to the graph you made in problem 5. Use a different color for each set of data.
a. Graph the data in your table from problem 7a. Write a description on your graph telling what assumption was used to obtain the numbers.
b. Repeat part (a) for the data in your table in problem 7b.
9. Compare the three graphs.
a. Which of the two assumptions in problem 7 gave a closer prediction of the population in 1900? How close was each estimate?
b. Which predicted the population in 1990 more accurately? How close was each estimate?

Say you were living in 1940, had access to the data for the period 1890-1940, and wanted to predict the population for 1950 and 1990.
10. a. Why might you not want to use the growth from 1930 to 1940 to help you make the predictions?
b. What numbers might you choose instead to model a constant number increase? What about a constant percent increase?
11. Repeat problem 7, starting with the 1940 population and using the numbers you chose in problem 10. Do you get better predictions?
12. Predict the population of the U.S. in the years 2000 and 2040. Explain how you arrive at your numbers.

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It is very expensive to conduct a census of the entire population, and it cannot be done every year. However, census data can be used to estimate the population in other years.
13. Use the 1940 and 1950 data to estimate the population in 1945 assuming
a. linear growth;
b. exponential growth.
14. Use the 1930 and 1950 data to estimate the population in 1940 assuming
a. linear growth;
b. exponential growth.
15. Use the 1890 and 1990 data to estimate the population in 1940 assuming
a. linear growth;
b. exponential growth.
16. Compare your answers to problems 14 and 15. Did you get closer to the actual 1940 population using
a. the 1930 and 1950 data or the 1890 and 1990 data?
b. the linear model or the exponential model?

## FTxivixe xuturitys

Definitions: When we know data points and use them to determine data values between those points, the process is called interpolation. When we know data points and try to use them to predict data values at a later or earlier time, the process is called extrapolation.

## 12.1

17. Which of the problems in this lesson involved extrapolation? Which ones involved interpolation?

It is important to examine assumptions when analyzing from data. People who analyze data often make incorrect projections and draw wrong conclusions because of making inappropriate assumptions.
18. Report Write a report summarizing what you learned in this lesson. Your report should include but not be limited to comments on:

- the suitability of the linear and exponential models as applied to the growth of the U.S. population during this century;
- the validity of results from extrapolating and interpolating using these models;
- a comparison of the accuracy of shortterm and long-term predictions;
- how ideas outside of mathematics can help improve the quality of a mathematical model.


## REVIEH IINE THROUGH TWO POINTS

19. Find the equation of the line through:
a. $(0,0)$ and $(12,34)$;
b. $(5,6)$ and $(7,11)$;
c. $(8.9,-10)$ and $(12.3,-4.3)$.
