LESSON 11.8

Unit Conversion

You will need:

graph paper 

1. Find the missing numerator and denominator in each equation. (You should be able to solve most of these without multiplying or dividing.) Compare your answers with other students' answers.

   a. $\frac{2}{3} \cdot \frac{5}{7} \cdot \frac{\triangle}{\square} = \frac{2}{3}$

   b. $\frac{2}{3} \cdot \frac{\triangle}{7} \cdot \frac{5}{\square} = \frac{2}{3}$

   c. $\frac{7}{12} \cdot \frac{14}{11} \cdot \frac{12}{5} \cdot \frac{\triangle}{\square} = \frac{7}{11}$

   d. $\frac{1}{3} \cdot \frac{5}{x} \cdot \frac{2}{5} \cdot \frac{x}{2} \cdot \frac{\triangle}{\square} = \frac{8}{3}$

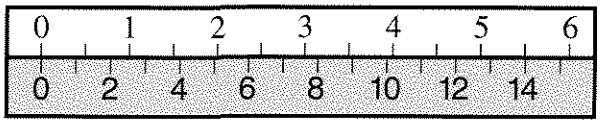
   e. $\frac{8}{7} \cdot \frac{3}{x} \cdot \frac{7}{6} \cdot \frac{y}{\square} \cdot = \frac{y}{x}$

   f. $\frac{a}{b} \cdot \frac{\triangle}{a} \cdot \frac{b}{\square} = \frac{x}{y}$

TWO RULERS

Alice had a new ruler. Oliver suggested she measure it with another ruler, as in this figure.

Alice's ruler



Oliver's ruler

Oliver and Alice had to write about functions for algebra. They decided to use the rulers as a way to get tables of x- and y-values. Here are the tables they got from the ruler setup.

Alice

x	y
1	2.5
2	5.1
3	7.6

Oliver

x	y
1	0.4
2	0.8
3	1.2

2. Describe the pattern for the numbers in each table.

3. What do you think the units of each ruler are?

4. Write a function of the type $y =$ *an expression in terms of* x for each table. (Because of measurement error, this may have to be an approximation.)

5. If you were to graph these functions, explain why the graph would
   a. be a line;
   b. pass through the origin.

6. According to an almanac, 1 inch = 2.54 centimeters, exactly. Using that information, what is the exact length of a centimeter, in inches?

When converting inches to centimeters, we multiply by 2.54. When converting centimeters to inches, we multiply by 1/2.54. As you can see from the equations, this conversion of units involves direct variation.

Definition: In the case of unit conversion, the proportionality constant (the number you multiply by) is called the *conversion factor*.

Conversion factors have units. For example, the conversion factor from inches to centimeters is 2.54 *cm/in*.

7. What is the conversion factor from centimeters to inches? (Include its unit.) Explain.

MULTIPLYING BY ONE

When converting a quantity from one unit to another, the way the quantity is measured is changed, not the amount of it. We can think of the conversion factor as having the value 1.

Example: Two miles are how many feet?

$$2 \text{ miles} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = 10{,}560 \text{ feet}$$

The conversion factor is 5280/1 and its units are feet/mile. Since 5280 feet = 1 mile, the numerator equals the denominator in the fraction, so we can think of this conversion as multiplying by a form of 1. To make the units work out, we multiplied by feet and divided by miles.

In problems 8-10, when writing a conversion factor, include its unit.

8. a. What is the conversion factor used to convert feet to miles?

b. Mount Everest, the world's tallest peak, is 29,028 feet high. How many miles is that?

9.

a. What is the conversion factor used to convert seconds to minutes?

b. What is the conversion factor used to convert minutes to seconds?

c. How are the answers to (a) and (b) related? Explain.

10. Convert 1000 inches to:

a. feet; b. miles;

c. meters; d. kilometers.

TWO-STEP CONVERSIONS

In science, speeds are sometimes given in feet per second. To convert feet per second to miles per hour, there are two steps:

- Convert feet to miles.
- Convert seconds to hours.

The steps can be combined:

$$\frac{\text{feet}}{\text{second}} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} \cdot \frac{3600 \text{ seconds}}{1 \text{ hour}} = \frac{\text{miles}}{\text{hour}}$$

We chose the conversion factors in order to divide by feet and multiply by seconds so that those units did not appear in the final answer.

11. Convert the speed of sound in cold water (4938 feet per second) to miles per hour. Show your calculations.

12. To convert feet per second to miles per hour, what single number could you multiply by? Explain how you obtained this conversion factor.

13. Find the conversion factor between each of these common measures of speed. Show all your work. Summarize your results in a table like this one. Give approximations to the nearest thousandth. (Note: m/sec means meters per second.)

		To:			
		mi/hr	km/hr	m/sec	ft/sec
From:	mi/hr	—	—	—	—
	km/hr	—	—	—	—
	m/sec	—	—	—	—
	ft/sec	—	—	—	—

14. In your table, find pairs of numbers that are reciprocals of each other. Explain why they should be reciprocals.

15. Use your table to convert
   a. the speed of light (299,792,500 m/sec) into miles per hour;
   b. the speed of sound in cold air (1,088 ft/sec) into miles per hour.

16. A fast runner can run a mile in four minutes. How fast is that in miles per hour?

17. **Project** Find out how fast students in your class walk, skip, run, move backwards, etc., by timing how long it takes them to cover a measured distance. Convert the speeds to miles per hour.

18. **Project** Find out how fast cars drive on a nearby street or road, by timing how long it takes them to cover a measured distance. Convert the speeds to miles per hour.

REVIEW ***SOLVING SYSTEMS***

Solve each system. Check first to see if you can tell that the system has no solution or an infinite number of solutions.

19. $\begin{cases} 2x + 6 = 3y \\ 4y = 12 - 3x \end{cases}$

20. $\begin{cases} -m - b = 25 \\ -m + b = 13 \end{cases}$

21. $\begin{cases} 2r + 2s = 60 \\ r - 2s = 5 \end{cases}$

22. $\begin{cases} 2m + n = -1 \\ m + 3n = -18 \end{cases}$

23. $\begin{cases} r - s = 1 \\ r + 3s = -11 \end{cases}$

24. $\begin{cases} \frac{2}{3}x + \frac{2}{5}y = 4 \\ x - 2y = 5 \end{cases}$

25. $\begin{cases} y = \frac{3}{7}(x - 8) \\ y - 4 = \frac{3}{7}(x + 6) \end{cases}$