LESSON 11.5

Dice Games

You will need:

dice 

TWO GAMES

1. **Exploration** Play these two games with another person. To play a game, roll a pair of dice 20 times. After each roll, add the numbers on the uppermost faces. Keep track of how many rolls each player wins. (See below.) Whoever wins the most rolls, wins that game.

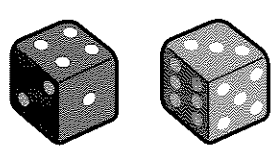
   Game One: If the sum is 3, 5, 7, 9, or 11, Player A wins. If the sum is 2, 4, 6, 8, 10, or 12, Player B wins.

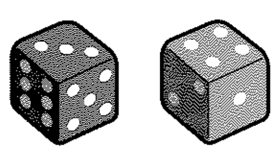
   Game Two: If the sum is 5, 6, 7, 8, or 9, Player A wins. If the sum is 2, 3, 4, 10, 11, or 12, Player B wins.

   For each game, who wins more often? Why?

TWO-DICE SUMS

If you roll a red die and a blue die, there are many possible outcomes. We will use (4, 3) to refer to the outcome in which 4 dots appear uppermost on the red die and 3 dots appear uppermost on the blue die. Likewise (3, 4) refers to 3 on the red die and 4 on the blue die.

(4, 3) 

(3, 4) 

Both of the outcomes in the figure show a sum of seven.

2. Copy and extend this table to show all possible two-dice sums. For each sum, list all the possible ways it can be obtained, and give the total number of ways. The sums of 2 and 7 have been done to get you started.

Sum	2	...	7	...	12
Possible ways	(1, 1)	...	(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)	...	...
# of ways	1	...	6	...	...

3. Which sums have the most ways of occurring? Which sums have the fewest ways of occurring?

4. **Summary** Analyze the games in problem 1 using the table you made. Explain why some sums are more likely to occur than others and how this determines who wins more often.

Definition: A game is *fair* if each of the players is equally likely to win.

5. Is Game One fair? How about Game Two? Explain.

OUTCOMES AND EVENTS

Definition: We call one roll of the dice an *experiment.* Each of the different possibilities you listed in the table is called an *outcome* of the experiment.

6. When you roll a red die and a blue die, how many outcomes are possible?

7. If you flip a penny and a nickel, how many outcomes (heads and tails) are possible? Make a list.

8. If you roll a red, a blue, and a yellow die, how many outcomes are possible?

When an experiment is performed, we are usually interested in whether or not a particular *event* has occurred. An event consists of one or more outcomes.

In the two-dice experiment, an example of an event could be: *The sum of the dots is even.* This event was important in Game One of problem 1. In that game, 36 outcomes were possible. However, we were not interested in the individual outcomes, but only in which of the two events had occurred: an even sum or an odd sum.

9. In what events were we interested in Game Two of problem 1?

10. The outcome of a two-dice experiment is (3, 2). Which of the following events occurred?
   a. The difference is even.
   b. The product is even.
   c. One die shows a multiple of the other.
   d. The sum is a prime number.

The table you made in problem 2 was organized to show these *events*: the sum of the dots is 2, the sum of the dots is 3, etc. In that table, each column corresponds to one event. A table like the following one is another way to represent the two-dice experiment. It is organized around the *outcomes*. Each cell corresponds to one outcome.

Blue Die

Red Die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
...	...	...	...	...	...	...

In the two-dice experiment, figure out how many outcomes make up each event in problems 11-14.

You can make the same kind of table to help answer problems 11-14. For example, to think about problem 11a, you would write the products in the cells.

Blue Die

Red Die	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	...	...	...	...

11. a. The product is even.
   b. The difference is even.
   c. One die shows a multiple of the other.

12. a. The sum is 2, 3, or 4.
   b. The sum is 9, 10, or 12.

13. a. a double
   b. not a double

14. a. The sum is prime.
   b. The product is prime.
   c. The difference is prime.

CREATE DICE GAMES

15. Name two events in the two-dice experiment that each consist of nine outcomes.

16. Name an event in the two-dice experiment that consists of:

a. 17 outcomes;

b. 19 outcomes.

17. Create a dice game that is fair. Write the rules. Then write an explanation of why the game is fair.

18. Create a dice game that appears to favor one player, but that actually favors the other. Or, make up a dice game that appears to be fair, but that actually favors one player. Write the rules and an explanation of the game.

DISCOVERY *THREE QUANTITIES, THREE CONSTRAINTS*

These problems were invented by algebra students. You may want to use colored slips of paper to solve them. In each one, there are three unknown quantities and three constraints. Try to find the three unknown quantities.

19. The red and yellow marbles add up to 5.
Blue and red add up to 7.
There are 8 yellows and blues altogether.

20. The blue and red add up to 9.
There are two times as many yellows as blues.
There are 15 marbles altogether.

21. Blue + 9 = Black
Blue times 3 = Red
Black + 1 = Red

22. $A + B = 11$
$A + C = 7$
$B + C = 6$
How many of each?