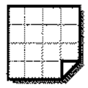
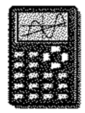
LESSON 9.5

The Square Root Function

You will need:

graph paper 

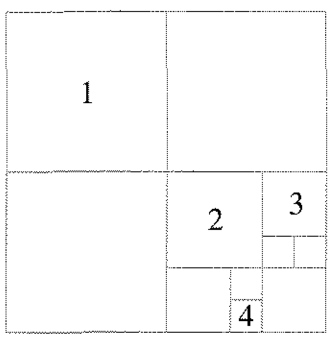
graphing calculator (optional) 

ROOTS OF NUMBERS < 1

The large square in the figure has dimensions 1-by-1 unit. It is divided into 11 smaller squares. For the square on the top left, you could write the following equations, relating the length of its side to its area.

a. $1/4 = (1/2)^2$ b. $1/2 = \sqrt{1/4}$

c. $0.25 = 0.5^2$ d. $0.5^2 = \sqrt{0.25}$



1. Explain the above equations.

2-4. For each numbered smaller square, write equations of the form:

a. area = side^2, using fractions

b. side = $\sqrt{\text{area}}$, using fractions

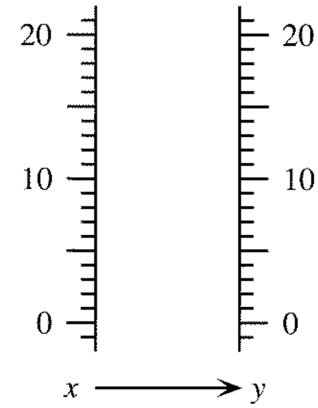
c. area = side^2, using decimals

d. side = $\sqrt{\text{area}}$, using decimals

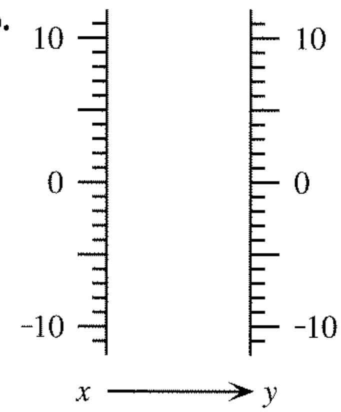
DIAGRAMS FOR SQUARES AND ROOTS

The function diagrams for the same function could look quite different with different scales.

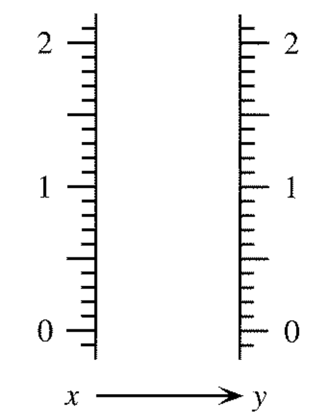
a.



b.



c.

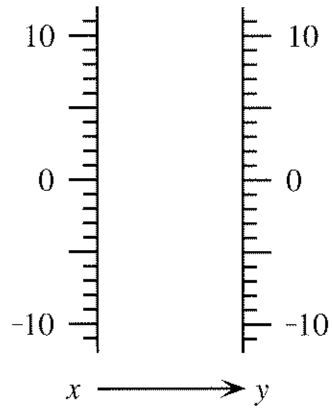


5. Make three function diagrams for the function $y = x^2$, using the scales given in the figure. Use nine in-out pairs for each.

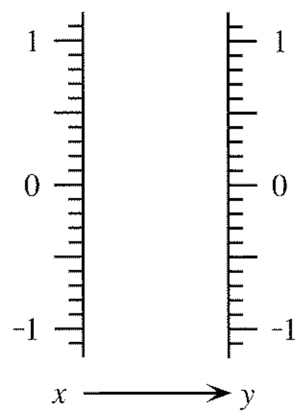
Problems 6 and 7 are about $y = x^2$.

6. In the function diagrams below, how far would you have to extend the y-number line in the positive direction so that every value you can see on the x-number line has a corresponding y-value on the diagram? How about in the negative direction?

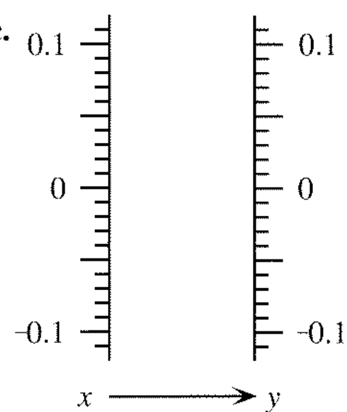
a.



b.

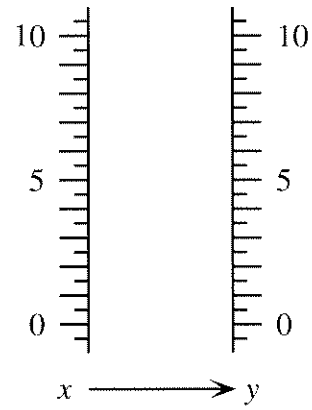


c.

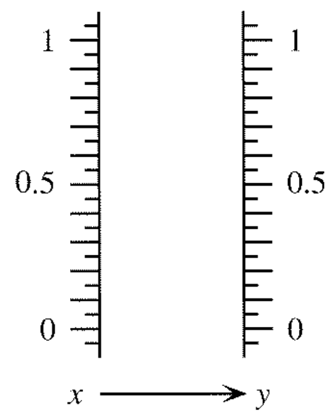


7. In the function diagrams below, how far would you have to extend the x-number line, if at all, so that every value you can see on the y-number line has a corresponding x-value on the diagram?

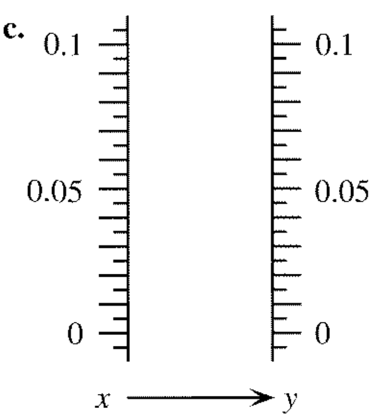
a.



b.



c.



Definitions: The *domain* of a function is the set of the values that the input can take. The *range* of a function is the set of the values the output can take.

Example: The domain of $y = x^2$ is *all numbers*, since any number can be squared.

8. Explain why the range of the function $y = x^2$ is all nonnegative numbers.

Notation: $\sqrt{x}$ represents the nonnegative number whose square is x.

Example: $\sqrt{4}$ represents only 2, even though $(-2)^2$ also equals 4. However we can write

$$-2 = -\sqrt{4}.$$

9. Using the same scales as in problem 5, make three function diagrams for the function $y = \sqrt{x}$.

10. For which scale is the function diagram not a mirror image of the corresponding one for $y = x^2$? Explain.

11. What are the domain and the range of the square root function? Explain.

12. To *be* or to *have*, that is the question.
a. Which numbers have a square root?
b. Which numbers have a square?
c. Which numbers can be a square?
d. Which numbers can be a square root?

GRAPHS FOR SQUARES AND ROOTS

13. Make tables of at least eight (x, y) pairs each for these two functions and graph them on the same axes. Use three values of x between 0 and 1, as well as negative values and whole numbers.
a. $y = x^2$ b. $y = \sqrt{x}$

14. On the same axes, graph the line $y = x$.

15. The curve representing $y = x^2$ is called a *parabola.* What would you call the curve representing $y = \sqrt{x}$?

16. Which of your three graphs grows
a. faster and faster?
b. more and more slowly?
c. always at the same rate?

17. If extended to the right, how high would the curve representing $y = \sqrt{x}$ go? (Can you find an x such that $\sqrt{x}$ is larger than 100? Than 1000?) Explain.

18.
a. What numbers are greater than their squares?
b. What numbers are less than their square roots?
c. What numbers are equal to their square roots?
d. What numbers are equal to their squares?

19. Solve the equations.
a. $x^2 = 5$ b. $x^2 = -5$
c. $\sqrt{x} = 5$ d. $\sqrt{x} = -5$
e. $-\sqrt{x} = -5$

20. Solve the inequalities. (Be careful! Some have compound solutions.)
a. $x^2 < 4$ b. $\sqrt{x} < 2$
c. $x^2 < \sqrt{x}$ d. $x^2 > 6$

21. Solve the equations and inequalities.
a. $P^2 = 456$ b. $P^2 < 456$
c. $\sqrt{K} = 789$ d. $\sqrt{K} < 789$

22. Report Summarize what you know about the functions $y = x$, $y = x^2$, and $y = \sqrt{x}$. Use graphs, diagrams, and examples. Include answers to these questions.
- Which is greatest and which is least among x, x^2, or $\sqrt{x}$? Explain how the answer depends on the value of x.
- What are the domains and ranges of these three functions?

23. Sketch the graphs of $y = \sqrt{x}$ and $y = \sqrt{-x}$. Think about domain and range!

MORE SQUARE ROOT GRAPHS

Use a graphing calculator if you have one.

24. Graph these equations on the same pair of axes.
    a. $y = 4\sqrt{x}$ b. $y = \sqrt{4x}$
    c. $y = \sqrt{4}\sqrt{x}$

25. In problem 24, which graphs are the same? Explain.

26. Graph these equations on the same pair of axes.
    a. $y = \sqrt{x + 9}$ b. $y = \sqrt{x} + 3$
    c. $y = \sqrt{x} + \sqrt{9}$

27. In problem 26, which graphs are the same? Explain.

PUZZLES *PACKING SQUARES*

28. A 10-by-10 square can be divided into 11 smaller ones, with no overlaps and no space left over (as in the figure at the very beginning of the lesson). Divide each of the following squares into 11 smaller squares. (The side lengths of the smaller squares must be integers.)
    a. 11-by-11 b. 12-by-12
    c. 13-by-13

DISCOVERY *WALKING DISTANCE*

Use graph paper as the map of a city. The horizontal and vertical lines represent streets.

29. In your group, agree on the location of various buildings, such as a supermarket, a hospital, a school, a fast food outlet, a bank, etc. Mark them on dot paper. Make a list of their coordinates.

    Make up a problem about finding a good place for a couple to live in your city. Assume that they do not want to drive, and that they work in different places. Each student should choose a different job for each member of the couple.
    a. Where should they live if they want to minimize the total amount of distance walked to work?
    b. Where should they live if, in addition, they want to walk equal amounts?