Essential Ideas

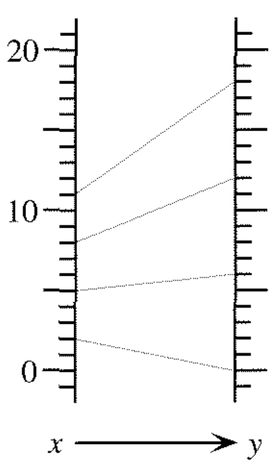
POPULATION OF NORTH AMERICA

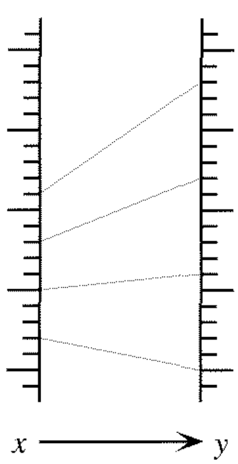
The table shows the estimated population of North America from 1650 to 1950.

Year	Population (thousands)
1650	5000
1750	5000
1850	39,000
1900	106,000
1950	219,000

1. What is the population increase in each 100-year period?
2. Graph the data.
   a. What is the meaning of slope for this data?
   b. Is the slope constant or does it increase or decrease? Explain.
3. Estimate the population of North America in the year
   a. 1800; b. 2000.
   Explain how you arrive at your estimates.

SAME DIAGRAM, DIFFERENT SCALE





4. Make an in-out table for the function diagram on the left. What is the function illustrated?
5. The function diagram on the right is the same, except that the number lines are not labeled. Copy the diagram, and put labels on it, using the same scale on both the x- and y-number lines. Make an in-out table, and find the function .
6. Repeat problem 5 two times. For each diagram, make an in-out table and find the function.

7. **Summary**
   a. For the functions you found in problems 4-6, when x increases by 1, what does y increase by? Does it depend on the scale you used?
   b. Compare the functions you found in problems 4-6. How are they the same? How are they different? Explain.

SLOPE AND INTERCEPT

The following questions are about the graph of $y = mx + b$.

8. Describe the line if $b = 0$ and
   a. $m > 1$ b. $0 < m < 1$
   c. $m = 0$ d. $-1 < m < 0$
   e. $m < -1$
9. In which quadrants does the line lie if
   a. $b > 0, m > 0$? b. $b < 0, m > 0$?
   c. $b > 0, m < 0$? d. $b < 0, m < 0$?
10. How would lines be the same or different if
    a. they have the same value for b and different values for m?
    b. they have the same value for m and different values for b?

LINEAR AND EXPONENTIAL GROWTH

11. Two populations are growing exponentially. At time 0, both have populations of 100. If one is growing twice as fast as the other, how do their populations compare after:

a. 2 hours? b. 3 hours?

c. x hours?

12. Report A recent college graduate was offered a job with a salary of \$20,000 per year and a guarantee of a 5% raise every year. She was about to accept the job when she received another offer for an identical job with a salary of \$22,000 per year and a guarantee of a \$1200 raise each year. Explain how you would help her decide which job to accept.

LAWS OF EXPONENTS

13. If possible, write as a power of 4.

a. $2 \cdot 2^6$ b. $(2 \cdot 2)^6$

c. $2 \cdot 2^5$ d. $2^7 \cdot 2^5$

e. $2^5 \cdot 2^5$

14. If possible, write as a power of 6.

a. $2 \cdot 3^5$ b. $(2 \cdot 3)^5$

c. 36^7 d. 36^0

15. If possible, write as a power of 3.

a. $9 \cdot 3^5 \cdot 3^2 \cdot 3^0$

b. $9 \cdot 3^5 \cdot 3^2 \cdot 2^0$

c. $9 \cdot 3^5 \cdot 2^2 \cdot 2^0$

d. $81 \cdot (3^5)^4 \cdot 6^0$

16. If possible, write as a single monomial.

a. $8a^{12} - 2(3a^3)^4$

b. $\left(\frac{6t^3}{4}\right)^2 - t^5$

17. Find values of a, b, and c so that

a. $(a \cdot b)^c > a \cdot b^c$;

b. $(a \cdot b)^c = a \cdot b^c$;

c. $(a \cdot b)^c < a \cdot b^c$.

18. Find the number or expression that makes each equation true. Write your answer as a power.

a. $(3x)^4 = \_\_\_\_ \cdot x^4$

b. $(5t)^3 = \_\_\_\_ \cdot t^3$

c. $(12xy)^3 = \_\_\_\_ \cdot (3xy)^3$

19. Simplify each ratio.

a. $(2x^5)/x^5$ b. $(2x)^5/x^5$

c. Explain why your answers to (a) and (b) are different.

20. Find the number that makes each equation true. Write your answer as a power.

a. $100 \cdot (2R)^5 = \_\_\_\_ \cdot 100 \cdot R^5$

b. $20 \cdot (2x)^7 = \_\_\_\_ \cdot 20 \cdot x^7$

c. $(2xyz)^{10} = \_\_\_\_ \cdot (xyz)^{10}$

21. Find the number that makes each equation true. Write your answer as a power.

a. $100 \cdot (3R)^5 = \_\_\_\_ \cdot 100 \cdot R^5$

b. $20 \cdot (3x)^7 = \_\_\_\_ \cdot 20 \cdot x^7$

c. $(3xyz)^{10} = \_\_\_\_ \cdot (xyz)^{10}$

22. Find the reciprocal. Check by showing that the product is 1.

a. $14x^3y^3$ b. $-3a^5$

c. $\frac{1}{3b^2}$

Because of variables in the exponents, these problems are more challenging.

23. Simplify.

a. $\frac{9 \cdot 10^{a+5}}{3 \cdot 10^a}$ b. $\frac{3 \cdot 10^{b+2}}{9 \cdot 10^b}$

c. $\frac{9 \cdot R^{a+5}}{3 \cdot R^a}$ d. $\frac{12 \cdot y^{b+2}}{10 \cdot y^b}$

24. Write as a power of 5.

a. $\frac{5^{2x-2}}{5^{x-5}}$ b. $\frac{5^{x-5}}{5^{2x-2}}$

25. Write as a power of 4. $\left(\frac{4^{3+x}}{4^{3-x}}\right)^3$

VERY SMALL NUMBERS

A proton weighs $1.674 \cdot 10^{-24}$ grams, an electron weighs $9.110 \cdot 10^{-28}$ grams, and Ann weighs 48 kilograms.

26. Which is heavier, a proton or an electron? How many times as heavy?

27. Ann weighs the same as how many

a. electrons?

b. protons?

28. The mean distance between the Earth and the sun is $1.50 \cdot 10^{11}$ meters. This length is called one *astronomical unit* (AU) and is a convenient unit for measuring distances in the solar system. The distance 10^{-10} meters is called one *angstrom* (after the Swedish physicist Anders Angstrom). It is a convenient unit for measuring atoms. How many angstroms are in one AU?