LESSON 8.12

Small and Large Numbers

1. Using a power of ten, write the reciprocal of each number.
   a. 10^2 b. 10^4 c. 0.001

SMALL NUMBERS IN SCIENTIFIC NOTATION

Any decimal number can be written in many ways as a product of a decimal number and a power of 10. For example, 43,000 can be written:

$$0.43 \cdot 10^5$$
$$4.3 \cdot 10^4$$
$$43 \cdot 10^3$$
$$430 \cdot 10^2$$
$$4300 \cdot 10^1$$

2. Write 43,000 as a product of a decimal number and
   a. 10^0; b. 10^{-1}; c. 10^{-2}.

3. a. Write 0.065 in three ways as a product of a decimal number and a power of 10. At least one way should use a negative exponent.
   b. Write 0.065 in scientific notation. (Remember that scientific notation requires multiplying a number greater than or equal to 1 and less than 10 by a power of 10.)

4. Which of these numbers would require a negative exponent when written in scientific notation? Explain why.
   0.0123 0.123 12.3 1230

5. How can you tell by looking at a decimal number whether or not it will require a negative exponent when it is written in scientific notation?

RECIPROCALS

Al and Abe, having nothing else to do, were arguing about reciprocals. Abe said, "If 10^{-4} is the reciprocal of 10^4, then $2.5 \cdot 10^{-4}$ is the reciprocal of $2.5 \cdot 10^4$." Al said, "I can prove that you're wrong by finding their product."

6. If $2.5 \cdot 10^{-4}$ is the reciprocal of $2.5 \cdot 10^4$, what should their product be?

7. Settle the argument between Al and Abe. If Abe has not found the correct reciprocal of $2.5 \cdot 10^4$, find it for him. Explain.

8. Find an approximation for the reciprocal of $4.6 \cdot 10^{-6}$. Give your answer in scientific notation.

UNITS AND RECIPROCALS

9. Dick walks at the rate of about five miles in one hour. What fraction of an hour does it take him to walk one mile?

10. Stanley can run about ten miles in one hour. What fraction of an hour does it take him to run one mile?

11. A snail travels at the rate of 0.005 miles per hour. How many hours does it take the snail to slither one mile?

Notice that your answers to problems 9-11 are the reciprocals of the rates given. This is not a coincidence. In each case, the rate is given in *miles/hour* and you are asked to find *hours/mile*. Since the units are reciprocals, the rates will also be reciprocals.

12. Sound travels through air at the rate of $1.088 \cdot 10^3$ feet per second at sea level. How long does it take sound to travel one foot?

13. Sound travels much faster through granite than through air. Its speed is about $1.2906 \cdot 10^4$ feet per second. How long does it take sound to travel one foot through granite?

UNITS IN THE METRIC SYSTEM

The metric system of measurement is based on powers of ten. Prefixes indicating powers of ten are used for all measurements within the metric system. Conversion between units is easy, since it involves multiplying by powers of ten.

Example: The prefix *kilo* means to multiply the basic unit of measure by 10^3, or 1000. A kilogram is 1000 grams, a kilometer is 1000 meters, and so on. This table lists some of these prefixes.

To Multiply by	Prefix
10^{12}	tera-
10^9	giga-
10^6	mega-
10^3	kilo-
10^2	hecto-
10^1	deka-
10^0	—
10^{-1}	deci-
10^{-2}	centi-
10^{-3}	milli-
10^{-6}	micro-
10^{-9}	nano
10^{-12}	pico-

14. Express the size of each object in terms of a more appropriate unit of measurement.

a. A redwood tree is 80,023 millimeters high.

b. A protozoan is 0.0000002 kilometers in diameter.

c. A football player weighs 95,130 grams.

15. At the San Andreas fault in Northern California, the ground is moving about $5 \cdot 10^{-5}$ kilometers per year. How long will it take to move one kilometer?

16. If hair grows at the rate of about 10^{-8} miles per hour, how long would it take your hair to reach ankle length? (Why is this problem harder than the previous ones?)