LESSON 7.10

Using Scientific Notation

WITH A CALCULATOR

Calculators can display numbers only up to a certain number of digits. For many calculators, ten digits is the limit.

1. What is the limit for your calculator?
2. What is the smallest power of 2 that forces your calculator into scientific notation?

On many calculators, the answer to problem 2 is 2^{34} which, according to the calculator, is equal to

1.717986918 [10] or 1.717986918E10 .

The expression on the left does *not* mean 1.717986918^{10}, even though that's what it looks like. It is just calculator shorthand for $1.717986918 \cdot 10^{10}$. The actual value is 17179869184, which is too long to fit, so the calculator gives the approximate value of 17179869180, expressed in scientific notation. (For a number this large, this represents a very small error.)

3. Which power of 2 is displayed as 2.814749767E14 ?
4. Find a power of 4 and a power of 8 that are also displayed as 2.814749767E14 .
5. Find powers of 3, 9, 27, and 81 that are displayed in scientific notation, in the form ____ $\cdot 10^{17}$. If possible, find more than one solution for each number.

There are three ways to enter numbers in scientific notation into your calculator. For example, to enter $2 \cdot 10^3$, you can key in 2 [*] 10 [^] 3, or 2 [*] [10^x] 3, or (depending on the calculator) 2 [EE] 3, or 2 [EXP] 3. We will refer to this last key as [EE] .

6. Try all the methods listed that are available on your calculator. In each case, the calculator should respond with [2000] after you press [=] or [ENTER].
7. Explain the purpose of the [^] and [EE] keys. How are they different?

HOW MUCH FARTHER, HOW MANY TIMES AS FAR?

The table shows the ten brightest objects in the sky, and their *average* distances from Earth, in miles. (The objects are listed in order of average brightness as seen from Earth.)

	Distance
Sun	$9.29(10^7)$
Moon	$2.39(10^5)$
Venus	$9.30(10^7)$
Jupiter	$4.84(10^8)$
Sirius	$5.11(10^{13})$
Canopus	$5.76(10^{14})$
Arcturus	$2.12(10^{14})$
Mars	$1.42(10^8)$
Vega	$1.59(10^{14})$
Saturn	$8.88(10^8)$

8. If you were to divide the objects into two groups, based only on the value of the exponents of 10, what would be in each group? What is the actual significance of the two groups?

For each pair of objects given in problems 9-13, answer questions (a) and (b). If an answer is greater than 10,000, give it in scientific notation.

a. The second object is *how many miles* farther from Earth than the first?

b. The second object is *how many times* as far from Earth as the first?

9. The Moon, Venus

10. The Moon, Saturn

11. The Sun, Sirius

12. The Sun, Canopus

13. Sirius, Canopus

WITHOUT A CALCULATOR

14. Convert these numbers to ordinary decimal notation and add them without a calculator.

a. $(4 \cdot 10^7) + (5 \cdot 10^6)$

b. $(40 \cdot 10^6) + (5 \cdot 10^6)$

15. Compare the two computations in problem 14. Which would have been easy to do without converting to ordinary decimal notation? Explain.

Without a calculator it is not easy to add and subtract in scientific notation. One way is to revert to ordinary decimal notation. Another is to write the two quantities with a common exponent for 10, as was done in problem 14b.

16. Add or subtract.

a. $6.2 \cdot 10^3 + 5 \cdot 10^6$

b. $6.2 \cdot 10^6 - 5 \cdot 10^3$

c. $6.2 \cdot 10^5 + 5 \cdot 10^3$

d. $6.2 \cdot 10^3 - 5 \cdot 10^6$

Without a calculator it can be tedious to multiply and divide large numbers. However, if the numbers are written in scientific notation it is easy to estimate the size of the answer.

For the following problems, 17-20:

a. Convert the numbers to ordinary decimal notation.

b. Multiply or divide.

c. Write your answers in scientific notation.

17. $(3 \cdot 10^5) \cdot (6 \cdot 10^3)$

18. $(3 \cdot 10^3) \cdot (6 \cdot 10^5)$

19. $(6 \cdot 10^6) \div (3 \cdot 10^3)$

20. $(3 \cdot 10^6) \div (6 \cdot 10^3)$

PREVIEW *MULTIPLICATION AND EXPONENTS*

21.

a. In each of problems 17-20, look for a relationship between your answer and the original numbers. How could you have obtained your answer without converting from scientific notation?

b. Explain a shortcut for multiplying and dividing numbers in scientific notation. Include an explanation of what happens to the exponent of 10.

22. Does the shortcut, described in problem 21b, work for multiplying $3(2^4)$ by $5(2^6)$? Explain, giving several examples of this type.

REVIEW *PERFECT SQUARE TRINOMIALS*

23. All of these are perfect square trinomials. Write each one as the square of a binomial.

a. $c^2x^2 + 2bcxy + b^2y^2$

b. $y^2 + 2xy + x^2$

c. $y^2 + 2by + b^2$

d. $0.25x^2 + 0.2x + 0.04$