Essential Ideas

CONSTANT SUMS AND PRODUCTS

1. If possible, write an equation of the form $x + y = S$ such that the graph of the equation
   a. lies in the 2nd, 3rd, and 4th quadrants;
   b. lies in the 1st, 2nd, and 3rd quadrants;
   c. passes through the origin;
   d. intersects the x-axis at (-7, 0);
   e. contains the point (12, -3.25).

2. A graph has an equation of the form $x + y = S$. Find two more points on the graph if:
   a. the point (-3, -5.8) is on the graph;
   b. the graph has x-intercept (1/2, 0);
   c. the graph has y-intercept (0, -6.5).

3. If possible, write an equation of the form $x \cdot y = P$ such that the graph of the equation
   a. lies in the 2nd and 4th quadrants;
   b. contains the point (-9, 1/2);
   c. passes through (-2.5, -3.5);
   d. intersects the graph of $x + y = 16$ at the point (10, 6);
   e. passes through the origin.

4. Write one equation of the form $x + y = S$ and one of the form $x \cdot y = P$ such that
   a. neither graph passes through the first quadrant;
   b. the two graphs intersect at (8, 4) and (4, 8).

THE DISTRIBUTIVE LAW

5. Write an equivalent expression without parentheses. Combine like terms.
   a. $2 \cdot (3 + x)$
   b. $2 \cdot (3x)$
   c. $(6x + 3)(2x - 4)$
   d. $(6x \cdot 3)(2x - 4)$
   e. $(6x \cdot 3)(2x \cdot 4)$

6. In which part of problem 5 did you use the distributive law to remove parentheses? Explain.

7. Write equivalent expressions without the parentheses. Combine like terms.
   a. $-2(9 + x) - x(2 - x)$
   b. $-2(9) + x - x(2 - x)$
   c. $-2(9 + x) - 2x - x$
   d. $-2(9) + x(-2x) - x$

8. In which parts of problem 7 did you use the distributive law to remove parentheses? Explain.

9. Write without parentheses. Combine like terms.
   a. $(x + 3)(x + 5)$
   b. $(x + 3)(x - 5)$
   c. $(x - 3)(x - 5)$
   d. $(x - 3)(x + 5)$

10. Divide.
   a. $\frac{6y^2 + 4xy}{2y}$
   b. $\frac{4x + 4}{4}$

FACTORING

11. Multiply $(2x - 7)(3x + 5)$.

12. Factor $6x^2 - 11x - 35$.

13. a. Fill in the blank with a whole number so that the trinomial $x^2 + 9x + \_\_$ can be factored as a product of binomials. Write the factored form.
   b. How many different integer answers are there for part (a)? Find all of them. (Don't forget negative integers.)

14. a. Fill in the blank with an integer so that the trinomial $x^2 + \_\_\, x + 18$ can be factored as a product of binomials. Write the factored form.

b. How many different integer answers are there for part (a)? Find all of them. (Don't forget negative integers.)

15. Factor completely.

a. $(2x + 8)(x^2 + 2x)$

b. $2yx^2 + 12yx + 16y$

c. $x^3 + 6x^2 + 8x$

16. How many x-intercepts does each parabola have? Explain.

a. $y = x^2 + 12x + 20$

b. $y = x^2 + 12x + 36$

c. $y = x^2 + 12x + 49$

d. $y = x^2 - 12x + 36$

17. In problem 16, find the coordinates of:

a. the y-intercept;

b. the x-intercept(s), if any;

c. the vertex.

SEQUENCES

18. If you were to plot these sequences (with n on one axis and t_n on the other axis), for which one(s) would the points lie in a straight line? Explain how you know.

a. 3, 3.5, 4.5, 5.5, 6.5

b. -1, -10, -19, -28, -37, -46

c. 1/2, 1/4, 1/8, 1/16, 1/32

d. 4, 7, 11, 16, 22, 29

PYRAMIDS

A pyramid is made by stacking rows of blue, red, and yellow blocks. There are 100 blocks in the bottom (first) row, 98 in the next row, and so on, with 2 fewer blocks in each successive row. The bottom row is blue, the next row is red, the third row is yellow, and so on, continuing the pattern.

19. Make a sketch or schematic drawing of what you think the pyramid might look like. Write about any patterns you notice.

20. How many rows of blocks are there?

21. How many rows of each color are there?

22. How many blocks are in the 10^{th} row? 11^{th} row? n^{th} row? Top row?

23. What color is the 10^{th} row? What color is the top row?

24. There are 30 blocks in a row. Which row is it?

25. Given the number of a row (5^{th}, 10^{th}, 20^{th}, etc.) can you give its color? Explain the pattern.

26. Given the number of blocks in a row, can you give its color? Explain the pattern.

27. How many blocks in all are needed to build the pyramid?

28. How would your answers to questions 19-27 be different if there were 50 blocks in the bottom row?

29. Suppose four colors were used instead of three. Would any of your answers to problems 19-27 be different? Explain.

30. **Report** Summarize and explain the patterns you noticed in the above problems. What generalizations can you make?