LESSON 5.8

Building-Block Numbers

In this lesson, use just whole numbers.

FOOD FOR THOUGHT

1. **Exploration** Eric tried to order 13 chicken nuggets at the fast food store. The employee informed him that he could order only 6, 9, or 20 nuggets. Eric realized he had to decide between ordering $6 + 6 = 12$, or $6 + 9 = 15$. What numbers of nuggets can be ordered by combining 6, 9, and 20? What numbers cannot be ordered? What is the greatest number that cannot be ordered? Explain.

TWO BUILDING BLOCKS

2. You have an unlimited supply of coins. What amounts can be obtained, and what amounts cannot be obtained using only dimes and quarters? Explain.

3. At Albert's Kitchen Supply, cabinets are available in two lengths: 3 feet and 5 feet. By putting cabinets end to end, walls of different lengths can be accommodated. Imagine that kitchens can be any size. What length walls are possible to line exactly with cabinets? What lengths are impossible?

4. In 1958 it cost 4 cents to mail a letter. In 1963 it cost 5 cents. Imagine you have an unlimited supply of 4- and 5-cent stamps. What amounts can you make? What is the largest amount you cannot make?

For each problem, 5-10, using only addition and the building-block numbers given, what numbers can you reach? What numbers can't you reach? If there is one, what is the greatest number you cannot reach?

5. 2, 5 **6.** 7, 11 **7.** 4, 6

8. 5, 12 **9.** 5, 15 **10.** 8, 1

Given the two numbers 7 and 11 and the operation of addition, it is possible to build every number beyond 59. However, with the numbers 4 and 6 there is no limit to the size of numbers that cannot be built.

11. **Generalizations** Suppose you find that for two numbers, *a* and *b*, and the operation of addition, you can build every number beyond a certain number. What can you say about *a* and *b*? Explain, using examples. (Hint: You may need to use the idea of common factors. For example: 4 and 6 have the common factor 2; 5 and 15 have the common factor 5.)

12. Given two numbers, *a* and *b*, such that their greatest common factor is 1, how can you calculate the greatest number that *cannot* be written as a sum of multiples of *a* and *b*? Explain, using examples.

A STRATEGY

In problems 13 and 14 you will investigate the numbers 5 and 6 as building blocks.

13. Write the numbers from 1 to 40 in an array like this.

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14...

a. Circle the multiples of 5. (0 is a multiple of 5.)

b. Circle the numbers that are equal to 6 plus a multiple of 5.

c. Circle the numbers that are equal to 12 plus a multiple of 5.

d. Circle the numbers that are equal to 18 plus a multiple of 5.
e. Circle the numbers that are equal to 24 plus a multiple of 5.

14. What is the largest number that **cannot** be built from 5 and 7? Explain how you know for sure that every number greater than this number can be built.

15. Repeat the same strategy to analyze 5 and 6 as building blocks.

16. Repeat the same strategy to analyze 4 and 7 as building blocks. (This time set up your array with only four columns.)

17. **Generalization** If you were to use the same strategy for numbers a and b, with $a < b$:

a. How many columns should you have in your array?
b. What numbers should you circle first?
c. What numbers should you circle next?
d. What is the smallest number in the last column you circled? (Write this number in terms of a and b.)
e. If you were not able to solve problem 12, try again with the help of this strategy.

DISCOVERY *HOLIDAY MATH*

18. Candles are lit every night for the eight nights of Hanukah. Two candles are lit on the first night, three on the second night, and so on, adding one candle each night. How many candles should be in the boxes of candles sold especially for Hanukah? Explain.

19. Find the words to the song "The Twelve Days of Christmas."

a. Make a sketch or drawing to show what is happening in the song. How many gifts did the singer receive on the twelfth day of Christmas? Explain.
b. The singer received six gifts on the 3^{rd} day. How many gifts did the singer receive on the 4^{th} day? The 5^{th} day? The n^{th} day? Explain.
c. The singer received 22 turtledoves. Find the total number of each other kind of gift that the singer received.
d. Suppose there were n days of Christmas. How many gifts would the singer receive in all? Explain.

PREVIEW *COIN PROBLEMS*

20. You have ten coins. Their total value is \$1.10. How many of each coin do you have? The problem has several solutions. Find as many as you can.

21. Add extra information that makes problem 20 have a unique solution. Explain how you know the solution is unique.

22. Create your own coin problem that has several solutions. Solve your problem.

23. Solve someone else's coin problem.

24. Add extra information to your problem so it will have a unique solution.

DISCOVERY **NEGATIVE STAMPS**

25. You want to mail a letter. It needs 52 cents postage, but all you have are 29-cent stamps: $29 + 29 = 58$. What would be convenient would be to have negative stamps. Then you could put two 29-cent stamps and a minus 6-cent stamp on your envelope, and it would solve your problem. Write a paragraph about this idea. How would the post office "sell" negative stamps? Why do you think they don't do it?

DISCOVERY **PAGE NUMBERS**

26. How many *digits* are used in numbering the pages of this book? Explain how you figured it out.

27. It took 1992 digits to number the pages of a book. Every page was numbered, starting with page 1. How many pages does the book have?

28. Explain how to find out how many digits are needed to number the pages of a book that has n pages, if n is
    a. more than 9, but less than 100;
    b. more than 99, but less than 1000.