LESSON 2.12

# Geoboard Triangles

You will need:

geoboards

dot paper

1. **Exploration** If many triangles have one vertical side in common, how is their area related to the position of the third vertex? To find out, make many triangles having vertices at (0, 0) and (0, 8). For each one, keep a record of the coordinates of the third vertex and the area. Look for patterns. Write a paragraph explaining what you found out. Use sketches.

## HORIZONTAL *AND* VERTICAL SIDES

2. Make a triangle having a horizontal side of length 6 and a vertical side of length 4. What is its area?

3. In this problem, use triangles having a horizontal side of 6.
   a. Make a table like the following. All triangles should have a horizontal side of length 6, but the length of the vertical side will vary. Extend the table all the way to vertical side of length 10.

| Vertical Side | Area |
|---|---|
| 0 | ... |
| 1 | ... |
| 2 | 6 |
| ... | ... |

   b. Explain how you could find the area c a triangle having horizontal side 6 anc vertical side 100.
   c. Express the area as a function of the vertical side.

4. Repeat problem 3 for a horizontal side o length 9.

## ONE HORIZONTAL *OR* VERTICAL SIDE

5. Make a triangle having vertices at (0, 0) and (0, 7) and the third vertex at (1, 4). What is its area?

6. Make a table like the following for triangles having vertices at (0, 0) and (0, 7) a the third vertex as indicated. Extend the table all the way to vertex (7, 4).

| 3rd Vertex | Area |
|---|---|
| (0, 4) | ... |
| (1, 4) | ... |
| (2, 4) | ... |
| ... | ... |

7. Write the area as a function of the $x$-coo dinate of the third vertex.

8. a. Make the triangle having vertices (0, (0, 7), and (9, 4). Guess its area.
   b. With another rubber band, make the smallest rectangle that covers the tria gle. If you did it correctly, you should now see two new triangles. Find the area of the rectangle and the area of t two new triangles.
   c. Find the area of the original triangle. This should match your guess from part (a).

**9.** How would you find the area of the triangle having vertices at (1, 0), (6, 0), and (9, 9)? Find it and explain what you did, using a sketch and a paragraph.

## Generalizations

**10.** a. Make triangles having vertices at (0, 0) and (0, 6) and the third vertex at $(x, 9)$, where $x$ takes each of the whole number values from 0 to 10. Make a table of values to show the area as a function of $x$.

b. Make triangles having vertices at (0, 0) and (0, 6) and the third vertex at $(9, y)$, where $y$ takes each of the whole number values from 0 to 10. Make a table of values to show the area as a function of $y$.

c. How do the answers to (a) and (b) differ?

**11.** a. Make at least three triangles having vertices at (0, 1) and (0, 6) and the third vertex at $(x, y)$, where $x$ and $y$ take whole number values from 1 to 8. Sketch each one and find its area.

b. Explain how you would find the area of a triangle having vertices at (0, 1), (0, 6), and (99, 99) without drawing a picture.

**12.** Explain how you would find the area of a triangle having vertices at (0, 0), $(b, 0)$, and $(x, h)$, where $b$ and $h$ are nonnegative.

## NO HORIZONTAL OR VERTICAL SIDES

**13.** **Exploration** What is the area of the triangle having vertices (0, 6), (7, 8), and (6, 1)? Explain how you arrive at the answer. Use sketches on dot paper.

**14.** What is the area of the four-sided shape having vertices at (0, 7), (2, 10), (10, 5), (5, 0)? Hint: First find the area of the whole geoboard, then use subtraction.

**15.** Make a triangle having no horizontal or vertical sides and having vertices on the outside edges of the geoboard. Use subtraction to find its area.

**16.** Repeat problem 15 on another triangle.

**17.** What is the area of the triangle having vertices at (1, 8), (2, 4), and (9, 3)? Hint: You may use the triangles having these vertices.

(1, 8), (1, 3), (9, 3)
(2, 4), (1, 3), (9, 3)
(1, 8), (2, 4), (1, 3)

**18.** **Report** Write an illustrated report on how to find the area of any geoboard triangle. Give examples of the different techniques. Make sure you include examples of using division by two, addition, and subtraction.