

## POLYOMINO EYES

Definition: The points of intersection of the grid lines inside a polyomino are called eyes.

1.

Exploration Any polyomino has an area, a perimeter, and a number of eyes. Is there a relationship between the three numbers? Can you express the perimeter as a function of the area and the number of eyes? (Hint: To find out, draw several polyominoes that have the same area, but different perimeters. For each one, write the number of eyes and the perimeter. As the number of eyes increases, does the perimeter get longer or shorter? Repeat the process for a different area.) Write a paragraph telling what you discover.
2. Complete the table shown at the top of the next column. Use data from these figures.
3. Write a formula for the perimeter of a polyomino having area 12 and $e$ eyes.


| Figure | Eyes | Area | Perimeter |
| :---: | :---: | :---: | :---: |
| a | 0 | 12 | $\ldots$. |
| b | 0 |  | .$\dot{ }$ |

4. Fill out a similar table for another area. Write a formula for the perimeter as a function of the number of eyes for your area.
5. If you know that a polyomino has 0 eye: and area 100 , how could you get its perimeter?
6. Answer question 5 using area 100 and 10 eyes.
7. Generalization Write a formula for the perimeter $p$ of a polyomino having area and $e$ eyes. (This formula is a function 0 two variables, $a$ and e.)
8. $\int$ For a given area, what is the maximu: number of eyes? Find a pattern by exper menting with areas 4 and greater.

## A GRAPH PAPER SPIRAL

9. Make a polyomino spiral on your graph paper by shading in one square at a time See the figure below.


Every time you shade a square, write the perimeter of the figure in a table like the following. Continue until you see a pattern.

| Area | Perimeter |
| :---: | :---: |
| 1 | 4 |
| 2 | 6 |
| 3 | 8 |
| ". | ". |

10. Describe the pattern you see.
11. Now make a new spiral the same way.

This time record only the areas of squares and rectangles that you get along the way, in two tables like those below, continuing until you see a pattern in all the columns.




| Square \# | Area | Perimeter |
| :---: | :---: | :---: |
| 1 | 1 | 4 |
| 2 | 4 | 8 |
| 3 | $\cdots$ | $\cdots$ |


| Rectangle \# | Area | Perimeter |
| :---: | :---: | :---: |
| 1 | 2 | 6 |
| 2 | 6 | 10 |
| 3 | $\cdots$ | $\cdots$ |

12. Describe the patterns you see in each column.
13. What will the area and perimeter be for square \#100?
14. Write a function for:
a. the area of square $\# x$,
b. the perimeter of square $\# x$.
15. What will the area and perimeter be for rectangle \#100?
16. Write a function for:
a. the area of rectangle \#x;
b. the perimeter of rectangle $\# x$.
17. Report What do you know about the relationship between area and perimeter of polyominoes? You may draw information from this lesson, as well as from Chapter 1, Lessons 1 and 2. Use graphs and illustrations.
