Making Sense in Algebra 2

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Lessons from a course developed over many years,
for somewhat heterogeneous classes,
with an emphasis on learning tools.

Presentation created with help from
Naoko Akiyama, Scott Nelson,
and other colleagues at
The Urban School of San Francisco
www.urbanschool.org

This packet includes what you will need during the workshop. Find links to many more worksheets, written for students, available for free download, at
<www.MathEducationPage.org/alg-2>.

Iterating Linear Functions: Special Cases

Definition: a fixed point of a function is one in which the output is the same as the input.

Example: For the function $y = 7x - 12$, when the input is 2, the output is also 2.

1. Find the fixed points:
   a. $y = 3x - 6$
   b. $y = 2x + 3$
   c. $y = 3x$
   d. $y = x$
   e. $y = x + 3$
   f. * $y = x^2 - 2$

2. There is a linear function that has more than one fixed point. What is it? Explain.

3. What linear functions have no fixed points? Explain.

When iterating a function, you get a sequence of numbers.

4. Start with the equation $y = 2x + 3$. Change one number in the equation so that when iterating the function, starting with any input, you get:
   a. an arithmetic sequence
   b. a geometric sequence
   c. a sequence where the values get closer and closer to a fixed point

Make a Rectangle, Make a Square

Make a rectangle with these blocks. For each rectangle, write an equation in the form "length times width equals area."

1. $x^2 + 7x$

2. $x^2 + 7x + 12$

Make a rectangle *with these dimensions*. For each rectangle, write an equation in the form "length times width equals area."

3. $x + 2$ and x

4. $x + 1$ and $x + 5$

Make a square with these blocks. One is impossible.

5. $x^2 + 2x + 1$

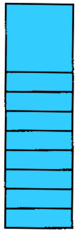
6. $x^2 + 4x + 4$

7. $x^2 + 6x + 8$

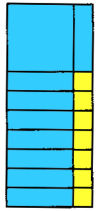
Make a square, with these blocks, plus some yellow blocks.

8. $x^2 + 6x + \dots$

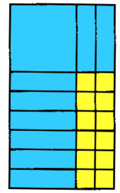
9. $x^2 + 10x + \dots$



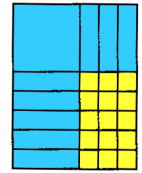
$(x + 8)x$
$= x^2 + 8x$



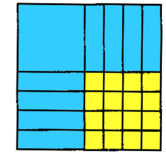
$(x + 7)(x + 1)$
$= x^2 + 8x + 7$



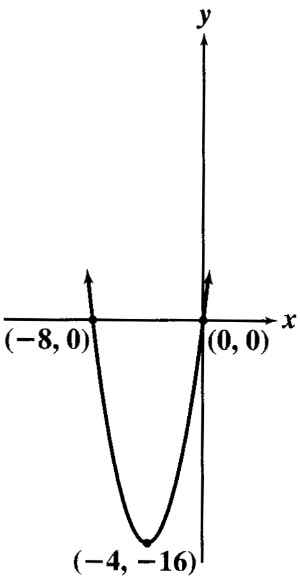
$(x + 6)(x + 2)$
$= x^2 + 8x + 12$



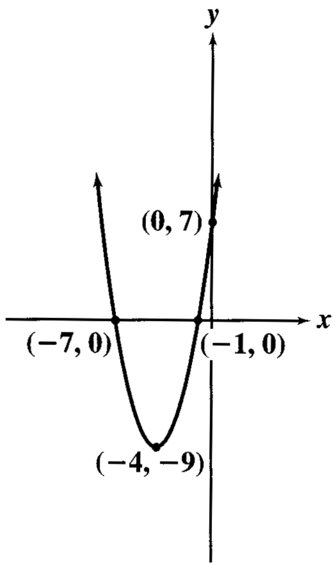
$(x + 5)(x + 3)$
$= x^2 + 8x + 15$



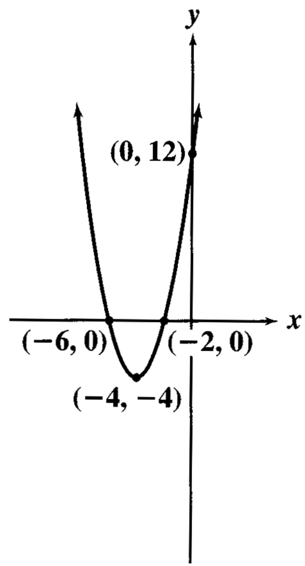
$(x + 4)^2$
$= x^2 + 8x + 16$



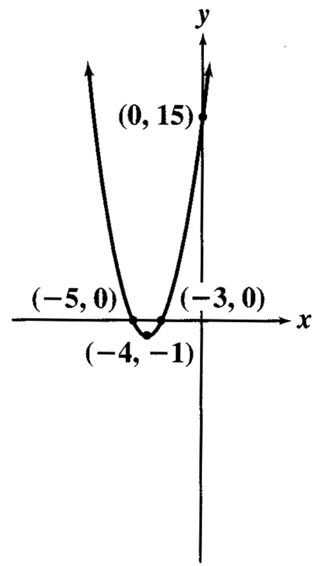
$y = x^2 + 8x$



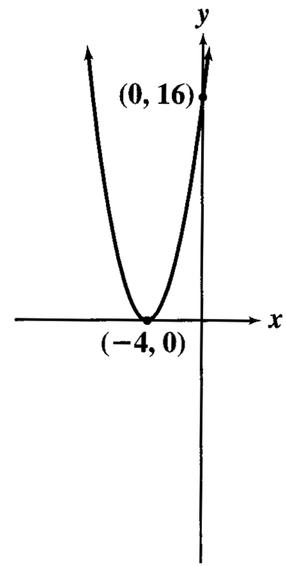
$y = x^2 + 8x + 7$



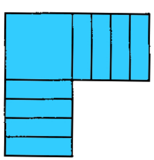
$y = x^2 + 8x + 12$

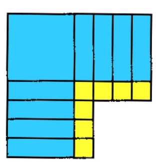


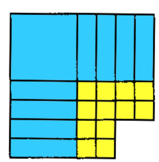
$y = x^2 + 8x + 15$

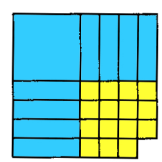


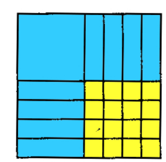
$y = x^2 + 8x + 16$











Math 3A, the Urban School of San Francisco Name:________________________

Rolling Dice

1. Roll forty 10-sided dice, and remove the dice that came up with a 0. Repeat this over and over. Record the results in the second column below:

How many dice are left?

How many rolls	Your experiment	Class average	Theory
0	40	40	40
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

Rolling Dice (cont)

2. Fill out the “class average” column.

3. Enter the class average data in your calculator, using STAT, EDIT. Graph them using STAT PLOT.

4. Fill out the “theory” column, by figuring that on average, about 10% of the dice get removed each time. Round the numbers to the nearest whole number.

5. Write an equation for f(x), the theoretical number of dice left after x rolls. Graph the function on your calculator, and check that it is a good model for the data.

6. What is the independent variable for f(x)? What is the dependent variable?

7. What are the domain and range of f(x)?

8. Is f(x) continuous or discrete?

9. Does f(x) have a y-intercept? What is its significance?

10. Does f(x) have an x-intercept? What is its significance?

Because x is in the exponent, this function is called an *exponential function*. You will learn much more about exponential functions in this course.

Super-Scientific Notation

Scientific notation:

$1200 = 1.2(10^3)$
$400{,}000 = 4.0(10^5)$

Super-scientific notation: 1200 and 400,000 can be written *as powers of ten*. We will call this *super-scientific* notation.

1. Explain why 1200 must be a power of ten with the exponent between 3 and 4.
2. 400, 000 must be a power of ten with the exponent between what whole numbers?
3. Find the power of ten that approximately equals the following numbers. Your answer should be accurate to the nearest thousandth.
   a. 1200 (Hint: start with a window with xmin=3, xmax=4, ymin=1000 and ymax=1400. Solve $10^x = 1200$.)
   b. 400,000

Looking for Patterns—Use Chart next pg.

4. Write the following numbers in super-scientific notation (with the exponents rounded to the nearest thousandth). Arrange the results in a table. Look for patterns as you work. Share the calculations with other students.
   a. The whole numbers from 1 to 9
   b. The multiples of 10 from 10 to 90
   c. The multiples of 100 from 100 to 900
5. What is the relationship between the exponents for 2, 20, and 200? Explain.
6. What is the relationship between the exponents for 3 and 9? Explain.
7. What is the relationship between the exponents for 20, 30, and 600? Explain.
8. What is the relationship between the exponents for 2 and 8? Explain.
9. Find other relationships between exponents, and explain them.

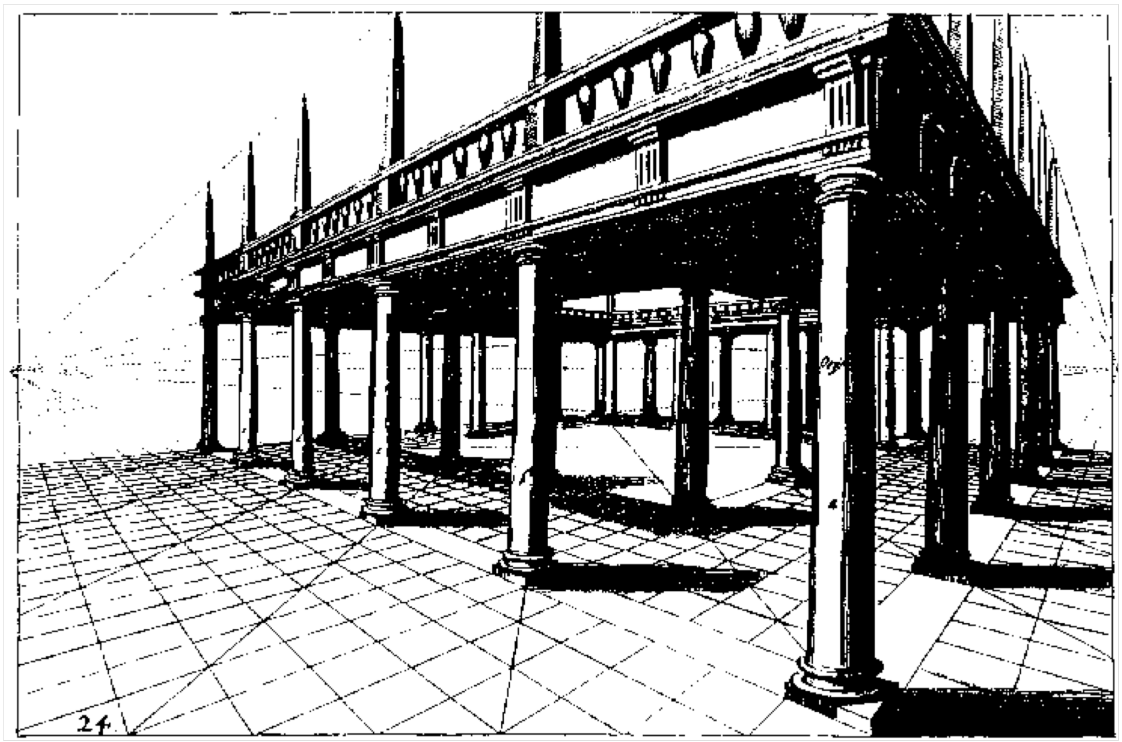
Calculating Without a Calculator!

10. Without a calculator, write the following in super-scientific notation. (Hint: use your table.)
    a. 9000
    b. .8
    c. .02
    d. 500,000
    e. 72
    f. 2/3
    g. 3/2
    h. 2700

Reflecting

11. Here is a calculation that uses scientific notation:
    $1200 \cdot 400{,}000 = 1.2(10^3) \cdot 4.0(10^5) = 4.8(10^8)$
    What is the equivalent calculation using super-scientific notation?
12. Explain the following statement: multiplying two numbers written in scientific notation involves a multiplication and an addition.
13. What is the corresponding statement for multiplying two numbers written in super-scientific notation? Explain.

Number	SSN	Exponent in SSN	Number	SSN	Exponent in SSN	Number	SSN	Exponent in SSN
1			10	10^1	1	100		
2	$10^{0.301}$	0.301	20			200		
3			30			300		
4			40			400		
5			50			500		
6			60			600		
7			70			700		
8			80			800		
9			90			900		



If the front pillar is 15 meters away, how far is the back pillar on the left?

Links and Resources

You will find more about all these topics on my Web site, on the page "Seeking Depth in Algebra 2":

<www.MathEducationPage.org/alg-2>

In addition:

Iterating Linear Functions

Activities on iterating linear functions by Jonathan Choate and Henri Picciotto, *The Mathematics Teacher*, February 1997.

Quadratics

The manipulatives approach to completing the square, and the connection to graphing, are developed in *Lab Gear Activities for Algebra 1*.

Rolling Dice

My Web site includes a version of the worksheet using Fathom software instead of the calculator.

Ten-sided dice are available at games stores. A similar activity is of course possible with normal six-sided dice, or coins, or M&Ms, but ten-sided dice are particularly convenient in statistics classes, to experiment with sampling distributions.

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